

國立高雄大學 107 學年度研究所碩士班招生考試試題

科目：離散數學與資料結構

系所：資訊工程學系

是否使用計算機：否

考試時間：100 分鐘

本科原始成績：100 分

1. Consider the following program segments. How many times is the **print** statement executed in them?

(a) (4%) for $i := 0$ to 10 do

 for $j := i$ to 10 do

 for $k := j$ to 10 do

 print $i + j + k$

(b) (4%) for $i := 0$ to 10 do

 for $j := 0$ to $i-1$ do

 for $k := 0$ to $j-1$ do

 print $i + j + k$

2. $x, y \in \{1, 2, 3, 4, 6, 12\}$. Let R be the relation that $x R y$ if $x | y$.

(a) (4%) Determine whether R is reflexive, symmetric, antisymmetric, or transitive.

(b) (4%) Show the Hasse diagram of R .

(c) (4%) If a topological sorting algorithm is applied to the Hasse diagram of R , list all the resulting total orders.

3. How many integer solutions are there for

(a) (5%) $w + x + y + z = 12$, $w \geq 0$, $x \geq 1$, $y \geq 2$, $z \geq 3$.

(b) (5%) $w + x + y + z = 12$, $0 \leq w \leq 2$, $1 \leq x \leq 4$, $2 \leq y \leq 6$, $3 \leq z \leq 8$.

4. (10%) Use the Principle of Mathematical Induction to show that $n \in \mathbb{Z}$, $\forall n \geq 1$,

$$3 \sum_{k=0}^{n-1} 4^k = 4^n - 1.$$

5. (10%) Use a combinatorial argument to show that $\sum_{k=0}^n \binom{n}{k} \binom{n}{n-k} = \binom{2n}{n}$, $\forall n \geq 0$.

6. In the design of search engines, typically, the first step is to construct an index file for the documents collected. For example, given a document with the following content:

Mary Jordan is faster than Mark Lewis

An index file contains a data structure to assist the search for the documents that contain given keywords (for example, “Mark” is a keyword in this document). For all the keywords in this document:

(a) Please show how to generate the binary search tree. (5%)

(b) Please show how many comparisons to find the keyword “faster”. (3%)

(c) Please show how to generate the AVL tree. (7%)

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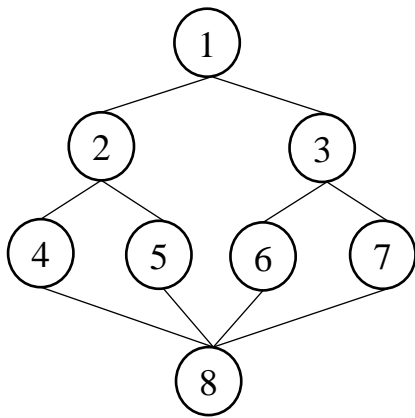
本科原始成績：100 分

(d) Please show how many comparisons to find the keyword “faster”. (3%)

7. Given the following graph:

(a) Please draw the linked list for each node in which the linked list includes only the first-level neighbors of each node (i.e., each node’s directly adjacent nodes). (5%)

(b) Then starting from node 5, use the linked lists generated from (a) to show how to print the value of each vertex by the Breadth-First Search method. (10%)



8. Is the following a correct equality? If yes, please prove it; otherwise, please show why it is incorrect. (5%)

$$n^3 2^n + 105n^2 3^n = O(n^3 2^n)$$

9. For two matrices **a** and **b** of size $n \times n$, please compute the time complexity of the following statements. (5%)

```
int i, j, k, sum;
for(i = 0; i < n; i++)
{
    for(j = 0; j < n; j++)
    {
        sum = 0;
        for(k = 0; k < n; k++)
            sum = sum + a[i, k] * b[k, j];
        c[i, j] = sum;
    }
}
```

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10. For the following B-tree of order 3, please show the results after deleting $\langle 5, 65 \rangle$. (7%)

