一. 是非題與選擇題（第 1-5 題為是非題，正確請填 T，錯誤請填 F，第 6-10 題為單選題）

1. (2%) If $f(x) = g(x)$ for all real numbers other than $x = 0$, and $\lim_{x \to 0} f(x) = L$, then $\lim_{x \to 0} g(x) = L$.

2. (2%) If f and g are increasing on an interval I, then fg is increasing on I.

3. (2%) If $f''(a) = 0$, then $y = f(x)$ has an inflection point at $x = a$.

4. (2%) If a point has polar coordinates (r, θ), then it also has polar coordinates $(-r, \theta + \pi)$.

5. (2%) If $f_x(a, b)$ and $f_y(a, b)$ both exist, then f is differentiable at (a, b).

6. (6%) Which one of the following series is convergent?

   A. $\displaystyle\sum_{k=1}^{\infty} \frac{\arctan k}{k}$　　B. $\displaystyle\sum_{k=2}^{\infty} \frac{k!}{k^k}$　　C. $\displaystyle\sum_{k=5}^{\infty} \left(\frac{\pi}{e}\right)^{k-1}$　　D. $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$

7. (6%) Evaluate the integral $\displaystyle\int_0^4 (x^2 + 1)e^{-x}\, dx$.

   A. $-27e^{-4} - 1$　　B. $27e^{-4} + 3$　　C. $-27e^4 + 3$　　D. $-27e^{-4} + 3$

8. (6%) Consider the function $f(x) = (x + 2)^{1/5} + 4$. Which one of the following statements is false?

   A. f is increasing on $\mathbb{R}$　　B. the graph of f has a vertical tangent line at $x = -2$　　C. the graph of f is concave up on $(-2, \infty)$　　D. f has an inflection point at $x = -2$

9. (6%) $\displaystyle\lim_{x \to 0^+} (\sin x)^x =$?

   A. 1　　B. 0　　C. e　　D. e^{-1}

10. (6%) Classify the quadratic surface $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$.

    A. Cone　　B. hyperbolic paraboloid　　C. hyperboloid of one sheet　　D. hyperboloid of two sheets

二. Find the limit.

1. (5%) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

2. (5%) $\displaystyle\lim_{x\to 0} \frac{x \csc 3x}{\cos 5x}$

三. Find the derivative.

1. (5%) $\dfrac{d}{dx} \ln(\tan^{-1} x)$

2. (5%) $\dfrac{\partial}{\partial y} \log_y x$

四. (10%) Let R be the annular region lying between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 3$. Evaluate the integral

$$\iint_R (x^2 + y)\, dA.$$

五. (10%) Evaluate the integral

$$\int_0^{\pi/2} \sin^7 \theta \cos^5 \theta \, d\theta.$$

六. (10%) Find the maximum and minimum values of $f(x,y,z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.

七. (10%) Find the principal unit normal vector to the curve $\mathbf{r}(t) = e^t \cos t\, \mathbf{i} + e^t \sin t\, \mathbf{j}$ at $t = \dfrac{\pi}{2}$.