國立彰化師範大學 100 學年度碩士班招生考試試題

系所：機電工程學系 　　　　　　　　　　　　　　　科目：工程數學

1. (25%)
   (a) Solve the following differential equation (15%)
   $$y''' - 5y'' + 8y' - 4y = e^{2x} + 2e^x + 3e^{-x}$$
   (b) Solve the following differential equation(10%)
   $$\frac{y'}{y^4} + \frac{1}{3y^3} = \frac{1}{3}(1 - 2x)$$

2. (25%)
   Solve the initial value problem for the system
   $$\begin{cases} y_1' - y_1 = 6u(t-2)e^{4t} \\ y_2' - y_1 - 2y_2 = 0 \end{cases}$$
   $$y_1(0) = 0, \, y_2(0) = 1$$
   Where *u(t-2)* is an unit step function or Heaviside function

3. (30%)
   (a).Explain The Green's theorem, Divergence theorem, and Stokes's Theorem **Mathematically.** (12%)
   (b).Explain the relation between the Green's Theorem and the Stokes's Theorem. (3%)
   (c).Explain and prove why the Green's theorem does not exist for the following double integrals? (15%),

   $$\iint_R (\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y})dxdy, where, F_1 = \frac{y}{x^2 + y^2}, F_2 = \frac{x}{x^2 + y^2},$$

   $$R: x^2 + y^2 = 1$$

4. (20%)
   Write the solution for the following system of linear equation in the form $X = X_h + X_P$ for the following equation, where X_h is the solution for AX=0 and X_p is a particular solution of AX=b , and X=$(x_1,x_2,x_3)^T$
   $$\begin{cases} x_1 + 3x_2 + 10x_3 = 18 \\ -2x_1 + 7x_2 + 32x_3 = 29 \\ -x_1 + 3x_2 + 14x_3 = 12 \\ x_1 + x_2 + 2x_3 = 8 \end{cases}$$