國立彰化師範大學 100 學年度碩士班招生考試試題

系所： 數學系　　組別： 乙組　　科目： 高等微積分

☆☆請在答案紙上作答☆☆　　

1. (10%) Let f be a real continuous function on $[a,b)$ $(a,b\in\mathbb{R})$. Suppose that f is differentiable on (a,b). If $\lim_{x\to a^+} f'(x) = A(\in\mathbb{R})$, then show that $f'(a)$ exists and $f'(a)=A$.

2. (15%) Let S be the set $\{(x,y)\in\mathbb{R}^2 | x>0, y>0, 2<xy<4 \text{ and } 1<x^2-y^2<9\}$. Compute $\int\int_S (x^2+y^2)dxdy$

   (Hint: change of variables: $u=x^2-y^2$, $v=2xy$)

3. (15%) Prove that the series $\sum_{n=1}^{\infty} \frac{\tan^{-1}(|x|^{\log n})}{n^{1.001}}$ defines a continuous function on $\mathbb{R}$.

4. (20%) Let f be a positive and continuous function on $[a,b]$ $(a,b\in\mathbb{R})$ and let $M=\max\{f(x)|x\in[a,b]\}$. Prove that $\lim_{n\to\infty}\{\int_a^b [f(x)]^n\}^{\frac{1}{n}} = M$.

5. (20%) Suppose that f is a function from a finite open interval (a,b) $(a,b\in\mathbb{R})$ into $\mathbb{R}$. If f is uniformly continuous on (a,b), then prove that f is bounded on (a,b); that is, there is an $M\geq 0$ such that $|f(x)|\leq M$ for all $x\in(a,b)$.

   (Hint: the closure of $(a,b)=[a,b]$, which is compact)

6. (20%) Prove or disprove that if f is a function from $\mathbb{R}$ into $\mathbb{R}$ and if $f(K)$ is compact for every compact subset K of $\mathbb{R}$, then f is continuous on $\mathbb{R}$.