國立成功大學 107 學年度碩士班招生考試試題

系　所：化學工程學系

考試科目：單元操作與輸送現象　　考試日期：0205，節次：1

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Equations

Navier-Stokes equation in cylindrical coordinates

r direction

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)$$
$$=-\frac{\partial P}{\partial r}+\rho g_r+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]$$

θ direction

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial P}{\partial \theta}+\rho g_\theta$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$$

z direction

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$
$$=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$

Shear stress components in cylindrical and spherical coordinates

$$\tau_{r\theta}=\tau_{\theta r}=\mu\left[r\frac{\partial}{\partial r}\left(\frac{\upsilon_\theta}{r}\right)+\frac{1}{r}\frac{\partial \upsilon_r}{\partial \theta}\right]$$
$$\tau_{z\theta}=\tau_{\theta z}=\mu\left[\frac{\partial \upsilon_\theta}{\partial z}+\frac{1}{r}\frac{\partial \upsilon_z}{\partial \theta}\right]$$
$$\tau_{zr}=\tau_{rz}=\mu\left[\frac{\partial \upsilon_z}{\partial r}+\frac{\partial \upsilon_r}{\partial z}\right]$$

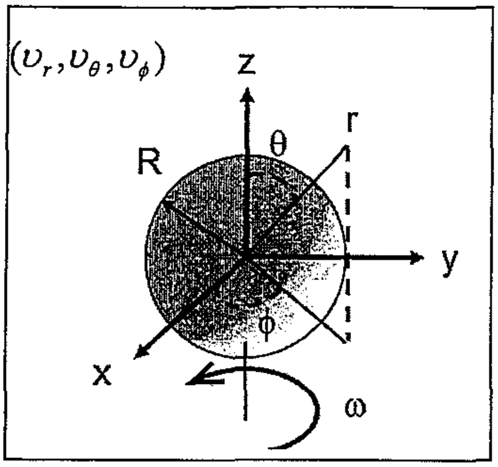
$$\tau_{r\theta}=\tau_{\theta r}=\mu\left[r\frac{\partial}{\partial r}\left(\frac{\upsilon_\theta}{r}\right)+\frac{1}{r}\frac{\partial \upsilon_r}{\partial \theta}\right]$$
$$\tau_{\phi\theta}=\tau_{\theta\phi}=\mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{\upsilon_\phi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial \upsilon_\theta}{\partial \phi}\right]$$
$$\tau_{\phi r}=\tau_{r\phi}=\mu\left[\frac{1}{r\sin\theta}\frac{\partial \upsilon_r}{\partial \phi}+r\frac{\partial}{\partial r}\left(\frac{\upsilon_\phi}{r}\right)\right]$$

Vorticity in spherical coordinates

$$\nabla\times\vec{\upsilon}=\left(\left(\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\upsilon_\phi\sin\theta)-\frac{\partial \upsilon_\phi}{\partial \phi}\right]\right)\vec{e}_r+\left(\frac{1}{r\sin\theta}\frac{\partial \upsilon_r}{\partial \phi}-\frac{1}{r}\frac{\partial}{\partial r}(r\upsilon_\phi)\right)\vec{e}_\theta+\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\upsilon_\theta-\frac{\partial \upsilon_r}{\partial \theta}\right)\right)\vec{e}_\phi\right)$$

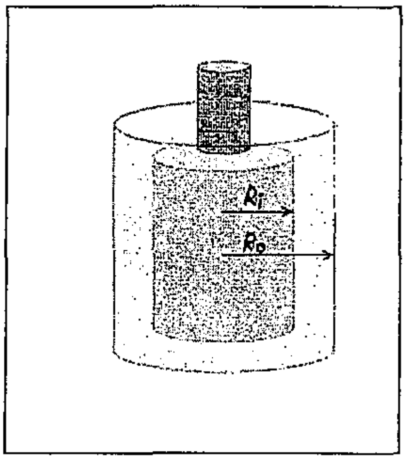
Questions

1. Consider a solid sphere of radius R rotating in a large body of a stagnant, incompressible Newtonian fluid as shown on the right. Assume that the fluid flow is laminar and steady. Assume no effect of gravity and pressure. The velocity was found to be $\vec{\upsilon}=(R^3\omega/r^2)\sin\theta\vec{e}_\phi$.



   (a) Derive the surface velocity at $\theta=45°$. (2%)
   (b) Derive all the components of the shear stress as a function of r and θ. (4%)
   (c) Derive the shear stress that the rotating sphere acts on the fluid at $\theta=45°$. Please specify the direction (4%)
   (d) Derive the vorticity on the sphere. (4%)

2. The right figure shows the geometry of a rheological experiment (**Stormer viscometer**). A fluid lies between an outer cylinder R_o (18.0 mm) and an inner cylinder R_i (17.0 mm). The inner cylinder is rotated at 4000 rpm. A torsion meter was installed on the outer cylinder and torque is measured to be 0.05 Nm. The length of the inner cylinder is 36 mm. Assume that the fluid is incompressible and Newtonian. And the viscous end effects are negligible. No-slip condition was applied on the solid surface.



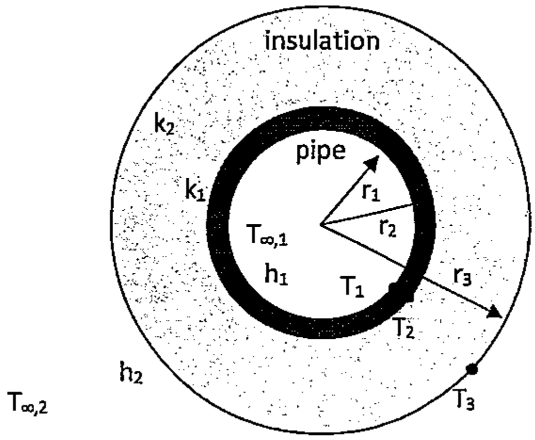
   (a) Write down the simplified Navier-Stokes equation and derive that the velocity is $\vec{\upsilon}=(C_1/r+C_2r)\vec{e}_\theta$, where C_1 and C_2 are constants. (6%)
   (b) Write down the boundary conditions and determine the constants, C_1 and C_2. (4%)
   (c) Explain how to determine the fluid viscosity and calculate it. (6%)

3. Explain the following: (10%)
   (a) Nusselt number
   (b) A lumped-parameter system (in unsteady state heat conduction)
   (c) Decrease of the heat flux after nucleate boiling during pool boiling
   (d) Heat exchanger effectiveness
   (e) Stefan-Boltzmann Law (in thermal radiation)

4. An insulated cylindrical pipe with length L is shown in the following. Consider one-dimension heat transfer through cylindrical layers,

(a) Derive the thermal resistance of the pipe. (7%)
(b) Find the critical radius of insulation. (8%)

The inner radius of the pipe: r_1
The outer radius of the pipe: r_2
The outer radius of the insulation: r_3
The thermal conductivity of the pipe: k_1
The thermal conductivity of the insulation material: k_2
The temperature of inner wall of the pipe: T_1
The temperature of outer wall of the pipe: T_2
The temperature of outer surface of the insulation: T_3
The temperature inside the pipe: $T_{\infty,1}$
The temperature outside the insulation: $T_{\infty,2}$
The heat transfer coefficient at inner surface of the pipe: h_1
The heat transfer coefficient at outer surface of the insulation: h_2

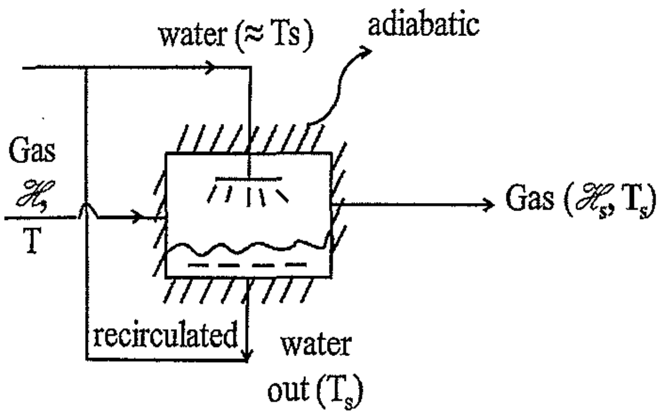


5. Two large tanks contain a mixture of N_2 and O_2, but at different concentrations. Tank A has 80 mole percent N_2 and 20 mole percent O_2, while Tank B has 15 mole percent N_2 and 85 mole percent O_2. The volumes of Tank A and Tank B are 20 and 40 m^2, respectively. A tube with a length of 2.0 m and an inside diameter of 0.10 m connects the two tanks. Assume that the pressure is 1 atm and the temperature is 25 °C.
   (a) By using the differential control-volume concept, derive the general expression for the flux of N_2 between the two tanks as a function of the partial pressure of N_2. List all the assumptions you made to obtain the N_2 flux. (12%)
   (b) The diffusion coefficient of N_2 in O_2 is 2.0×10^{-5} m^2/s at the given conditions. Calculate the rate of N_2 diffusion between the tanks. (4%)
   (c) By the use of the "film concept" for mass transfer, find the convective mass-transfer coefficient. (4%)

6. (a) In an adiabatic saturator shown below, a liquid was sprayed into a stream of gas. The input gas, with an initial humidity $\mathscr{H}$ and temperature T, was cooled and get equilibrium with the liquid. The exit temperatures of liquid and gas are all T_s (termed as *adiabatic saturation temperature*) and the humidity of the leaving gas becomes saturated ($\mathscr{H}_S$). The remaining liquid was recirculated to the spray nozzle. Please derive the following equation (8%)

$$\frac{\mathscr{H}_s - \mathscr{H}}{T - T_s} = \frac{C_s}{\lambda_s}$$

Where
C_s: humid heat capacity
λ_s: latent heat of water at T_s



(b) If the temperatures of the input (T) and output (T_s) gases can be measured, how to measure the **humidity** and **web-bulb temperature** of the input gas by humidity chart? Drawing a simple figure to describe your answer. (7%)

7. A gas stream contains 50% (in mole percent) acetone and 50% air. The acetone is to be removed by adsorption into a nonvolatile oil. The entering oil contains 1% acetone. The equilibrium relationship of acetone concentration in the air (y) and oil (x) is $y = 2x$. If one would like to remove 96% of the acetone from the entering gas, what is the minimum flow rate of the entering oil (L_a) per 100 mole of entering gas? What is the number of ideal stages required in this condition? (10%)