※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Based on the Boundary Value Problem (BVP), please explain (a) **Dirichlet** boundary condition and (b) **Neumann** boundary condition. **(10%)**

2. Solve the Ordinary Differential Equations (ODE) by the **Series method.** **(25%)**
$$(x^2 - x)y'' - xy' + y = 0$$

3. Find the response of the damped mass-spring system under a Unit Impulse at time $t = 1$. **(15%)**
$$y'' + 5y' + 6y = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 0$$

4. Compute the **minimum square error** E^* of $F(x)$ with $N = 1, 2$ and 3 relative to $f(x) = x + \pi$, $(-\pi < x < \pi)$ on the interval $-\pi \le x \le \pi$,

   where $F(x) = A_0 + \sum_{n=1}^{N} A_n \cos nx + B_n \sin nx$. **(20%)**

   (Hint: $f = f_1 + f_2$, $E^* = \int_{-\pi}^{\pi} f^2 dx - \pi \left[2a_0^2 + \sum_{n=1}^{N} (a_n^2 + b_n^2) \right]$, $\pi = 3.14$, $\pi^3 = 31.00$,

   E^*計算至小數點後第2位).

5. Find $y(x) = ?$. (hint: $u = Ax + By$). **(15%)**
$$\frac{dy}{dx} = (-2x + y)^2 - 7$$

6. Solve the nonhomogeneous ODE. **(15%)**
$$y'' + y = \sec x = \frac{1}{\cos x}$$