

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the general solutions of following ordinary differential equations.

a. $y'' + 4y' + 5y = 25x^2 + 13 \sin(2x)$ (10%)

b. $y'' + 2y' + 5y = 0$ (10%)

c. $x^3 dx + y^3 dy = 0$ (10%)

d. $9x dx + 4y dy = 0$ (10%)

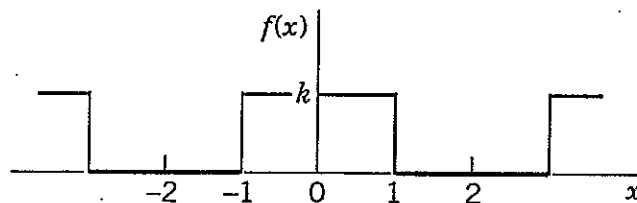
2. From an ordinary differential equation (ODE) of the form,

$$\frac{d^2x}{dt^2} + \omega^2 x = E_0 \cos(\omega_0 t)$$

Please describe the solutions and physical meanings with $\omega > 0$ and $\omega < 0$, respectively. (20%)

3. Find the Fourier series of the function

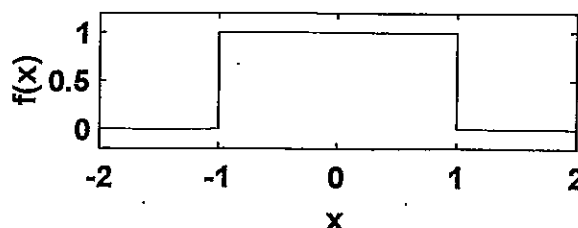
$$f(x) = \begin{cases} 0, & \text{if } -2 < x < -1 \\ k, & \text{if } -1 < x < 1 \\ 0, & \text{if } 1 < x < 2 \end{cases} \quad p = 2L = 4, L = 2$$



(15%)

4. Find the Fourier transform of

$$f(x) = \begin{cases} 1, & \text{if } |x| < 1 \\ 0, & \text{otherwise} \end{cases}$$



(15%)

5. Find the eigenvalue and eigenvector of the following matrix.

$$\begin{bmatrix} 85 & -28 & -28 \\ -10 & -11 & -11 \\ -46 & -2 & -2 \end{bmatrix}$$

(10%)