※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let X be a random variable and its moment generating function is $M_X(t) = \exp(e^t - 1)$.

   (1) Find the mean and variance of X. (10%)

   (2) Find the probability that $P(X > 1.5)$. (5%)

2. Let X and Y be random variables with joint density function

$$f(x,y) = \begin{cases} 1/x & \text{if } 0 < y < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (1) Compute $E(X)$. (5%)

   (2) Compute the conditional pdf of Y given $X = x$, for all $0 < x < 1$. (5%)

   (3) Compute Cov(X,Y). (5%)

3. Air quality deterioration has been a great concern in the recent years. A study monitors 10 metropolitan areas and find the number of days each year that the areas failed to meet acceptable air quality standards. The observed data are shown.

	City 1	City 2	City 3	City 4	City 5	City 6	City 7	City 8	City 9	City 10
Year 2016	50	23	120	40	35	28	99	60	43	31
Year 2017	55	20	135	47	42	30	108	68	50	26

Perform an appropriate hypothesis testing to see if the air quality does get deteriorated over time given $\alpha = 0.05$. (10%)

4. A researcher wishes to estimate the proportion of 12-year-old children who own a smart phone. Assume that there is no previous information regarding the proportion of 12-year-old children owning a smart phone. To be accurate within 2% of the true proportion with 95% confidence, find the minimum sample size. (10%)

※ 考生請注意：本試題可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

5. (10 %) Two radiographers were asked to inspect 105 sheets of chest x-ray images for screening potential tuberculosis infection. The radiographers (A and B) either classify the image as "Abnormal" (for further study) or "Normal" (No further study required). The results are as follows.

1) 36 images were rated Abnormal by both.
2) 53 images were rated Normal by both.
3) Moreover, Radiographer A rated Abnormal to 43 images and Normal to 65 images.

Please use the Kappa statistic proposed by Cohen to estimate the strength of agreement (inter-rater reliability) between the two radiographers.

6. (13%) One researcher wanted to know whether physical exercise can alleviate depression or not. First, he recruited some people interested in this study from the local community and had their depression level checked. Eighteen people with an equivalent level of depression were then randomly assigned to one of three groups: 40 minutes of muscle strength training per day, 40 minutes of treadmill running per day, or 40 minutes of yoga workout per day. At the end of three months, he asked each participant to rate how depressed they now feel on a Likert scale that runs from 0 ("not at all") to 100 ("completely miserable").

Please use the ratings results in the following table to determine whether these three types of physical exercise were equally effective? Significance Level (α) = 0.05.

Participant number in each group	Strength training	Treadmill running	Yoga workout
1	77	78	41
2	74	55	34
3	49	61	62
4	42	71	55
5	63	55	71
6	71	71	40

7. A method engineer wanted to investigate the effect of four different wrenches (A, B, C, D) on the task time for fastening drive engines. Four workers were selected for the study. The Latin square design was used and the recorded task times were provided in the following table. The alpha level was set at 0.05. Please analyze the experiment data and draw appropriate conclusion.

	Worker			
Order of experiment	1	2	3	4
1	10(A)	8(B)	9(C)	7(D)
2	6(B)	19(C)	8(D)	15(A)
3	12(C)	9(D)	10(A)	8(B)
4	10(D)	16(A)	12(B)	15(C)

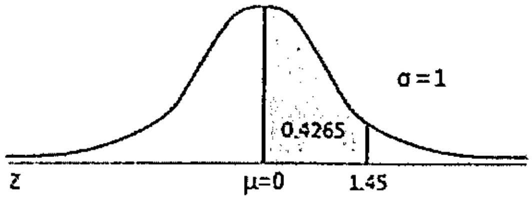
1) (13%) Is there a significant "Wrench" effect on the task time? Please provide the ANOVA table under this scenario.

2) (6%) Would the conclusion for the "Wrench" effect change if you analyzed the data using regular one-way ANOVA?

3) (8%) Describe the advantages and disadvantages of Latin square design

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

國立成功大學 107 學年度碩士班招生考試試題

系　所：工業與資訊管理學系

考試科目：統計學

考試日期：0206，節次：3

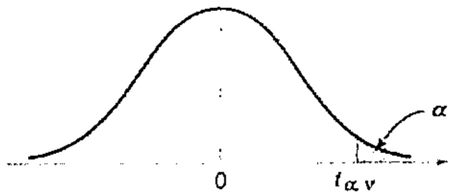


Table IV　Percentage Points $t_{\alpha,\nu}$ of the t-Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090

ν = degrees of freedom.

IV Percentage Points of the *F* Distribution (*Continued*)

$F_{0.05,\nu_1,\nu_2}$

Degrees of Freedom for the Numerator (ν_1); Degrees of Freedom for the Denominator (ν_2)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38