

國立高雄師範大學 100 學年度碩士班招生考試試題

(請用藍、黑色筆作答，以其他顏色或鉛筆作答者不予計分)

系所別：數學系

科 目：分析 (包括微積分、高等微積分、複變數函數) (第 1 頁, 共 2 頁)

1. Find $\lim_{x \rightarrow \infty} \left(\frac{b^x - a^x}{a^{x-1}x(b-a)} \right)^{1/x}$, where $a > 0$, $b > 0$, and $a \neq b$. (10%)

2. Show that $\int_2^{\infty} e^{-\frac{(x-2)^2}{3}} dx = \frac{\sqrt{3\pi}}{2}$. (10%)

3. Use the mean value theorem to show that $10\sqrt{x+3} < x+24$ if $x > 46$. (10%)

4. Define $f_n : [-1, 1] \rightarrow \mathbb{R}$ by $f_n(x) = \sum_{k=1}^n \frac{x^k}{k2^k}$, $\forall x \in [-1, 1]$; (a) Is $\{f_n\}_{n=1}^{\infty}$ uniformly Cauchy?

Give your reason. (b) If the answer of (a) is yes, does there exist a

function $f : [-1, 1] \rightarrow \mathbb{R}$ $\ni f_n$ converges to f uniformly? (10%)

5. Determine that if the improper integral $\int_0^{\pi/2} \ln \sin x dx$ converges? If this is yes, find its value. (10%)

6. Assume that $S \subset \mathbb{R}^n$ and S is compact (i.e. every open covering of S contains a finite subcover) Prove that S is closed and bounded in \mathbb{R}^n . (10%)

7. Let $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{\frac{2x_n}{x_n+1}}$. Prove that $\prod_{n=1}^{\infty} x_n = \frac{\pi}{2}$, where $\prod_{i=1}^n i = 1 \times 2 \times 3 \times \dots \times n$. (10%)

8. Let two functions f and g be analytic inside and on the simple closed contour $\Gamma \subset \mathbb{C}$.

Prove that if $f(z) = g(z)$ for all z on Γ , then $f(z) = g(z)$ for all z inside Γ . (10%)

(背面有題)

系所別：數學系

科 目：分析（包括微積分、高等微積分、複變數函數）（第 2 頁，共 2 頁）

9. Evaluate

$$\int_{\Gamma} \frac{e^{iz}}{(z^2+1)^2} dz$$

where Γ is the circle $|z|=3$ traversed once counterclockwise. (10%)

10. Compute

$$\int_{\Gamma} \frac{\cos z}{z^2(z-3)} dz$$

along the contour Γ indicated in the following figure: (10%)

