國立高雄師範大學 100 學年度碩士班招生考試試題

（請用藍、黑色筆作答，以其他顏色或鉛筆作答者不予計分）

系所別：數學系

科　目：代數（包括線性代數及代數學）（第 **1** 頁，共 **2** 頁）

1. Let R be the set of all real numbers and let C be the set of all complex numbers.
   (1) Let $V = \mathsf{C}^2, F = \mathsf{R}$, Is V is a vector space with the operations of coordinatewise addition and multiplication? Why?（5%）
   (2) Let $V = \mathsf{R}^2, F = \mathsf{C}$, Is V is a vector space with the operations of coordinatewise addition and multiplication? Why?（5%）

2. Let $V = \mathsf{R}^2$, Define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2)$ and $c(a_1, a_2) = (ca_1, a_2)$. Does the identity of addition exist? If exists, find it.（5%）

3. Let V and W be vector spaces and $T: V \to W$ be linear. Show that $R(T) = \{T(x) \mid x \in V\}$ is a subspace of W and $N(T) = \{x \in V \mid T(x) = 0\}$ is a subspace of V.（10%）

4. Suppose that $T: \mathsf{R}^2 \to \mathsf{R}^2$ is linear, $T(1,0) = (1,4), T(1,1) = (2,5)$. What is $T(2,3)$? Is T one-yo-one ?（5%）

5. Let $A, B \in M_{n\times n}(F)$ such that $AB = I_n$. Prove that A and B are invertible and $B = A^{-1}$.

   （10%）

6. If a<0, is $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ a & 0 & 0 & 0 \end{bmatrix}$ diagonalizable over R ? Why?（10%）

7. Let N be a normal subgroup of a finite group G.
   (1) If the order $|N|$ and the index $[G:N]$ are relatively prime, show that any element $g \in G$ satisfying $g^{|N|} = 1$ must be in N.（10%）
   (2) If H is a subgroup of G such that $|N|$ and $[G:H]$ are relatively prime, show that $N \subseteq H$.（10%）

（背面有題）

系所別：數學系

科　目：代數（包括線性代數及代數學）（第 **2** 頁，共 **2** 頁）

8. (1) Show that for all $n \geq 2$ and all primes p, $x^n - p$ is irreducible over $\mathbb{Q}$, the field of rational numbers.（10％）

   (2) Show that the degree of the field extension $\mathbb{C}$ over $\mathbb{Q}$ is infinite.（10％）

9. Determine all ring-homomorphisms from $\mathbb{Z} \oplus \mathbb{Z}$ to $\mathbb{Z}$.（10％）