

國立交通大學 107 學年度碩士班考試入學試題

科目：微積分與線性代數(4081)

考試日期：107 年 2 月 1 日 第 1 節

系所班別：統計學研究所 組別：統計所

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【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. 15% Let $g(x)$ be the inverse function of $f(x) = x \ln x$ for $x \geq 1$. Find $\int_0^e g(x) dx$.
2. 15% Find $\lim_{n \rightarrow \infty} \left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \dots + \frac{n}{n^2+n^2} \right)$
3. 20% Let $\phi(x)$ and $\Phi(x)$ denote the probability density function and the cumulative distribution of a standard normal distribution. Find $\lim_{x \rightarrow \infty} x \frac{1-\Phi(x)}{\phi(x)}$.
4. 25% If A is 3 by 3 symmetric positive definite, then $Aq_i = \lambda_i q_i$ with positive eigenvalues and orthonormal eigenvectors. ($\lambda_1 < \lambda_2 < \lambda_3$)
Suppose $x = c_1 q_1 + c_2 q_2 + c_3 q_3$
(a) Compute $x^T x$ and also $x^T A x$ in terms of c 's and λ 's.
(b) Consider $\frac{x^T A x}{x^T x}$, what c 's would make that ratio as large as possible?
5. 25% Let $C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
(a) Find the eigenvalues of C .
(b) What is C^{-1} ?
(c) Find the determinant of $C + 5I$.