國立交通大學 107 學年度碩士班考試入學招生試題

【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

請詳列計算過程，僅有答案沒有計算過程，將不予以計分。

First Part: Linear Algebra: (*50 points*)

1. Let $A \in M_{3\times3}(\mathbb{R})$ and $B \in M_{3\times3}(\mathbb{R})$ be two 3×3 real matrices such that

$$A(A-2B) = -(A-2B)A,$$

please answer the following questions:

(a) (8%) Show that $det(AB - BA) = 0$.

(b) (8%) If the matrix B is given as follows:

$$B = \begin{pmatrix} 2 & -2 & 1 \\ -1 & 3 & -1 \\ 2 & -4 & 3 \end{pmatrix},$$

find all possible matrices A satisfying (1).

2. Let $A \in M_{2\times2}(\mathbb{C})$ be a 2×2 complex matrix. We define the set $W(A) \subset \mathbb{C}$ as follows:

$$W(A) = \{v^*Av : x \in \mathbb{C}^{2\times1}, x^*x = 1\},$$

where $x^* \in \mathbb{C}^{1\times2}$ is the conjugate transpose of $x \in \mathbb{C}^{2\times1}$. Please answer the following questions:

(a) (8%) Given a matrix A as follows:

$$A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix},$$

compute the set $W(A)$?

(b) (8%) Given the set $E = \{z \in \mathbb{C} : |Re(z)| \le 1, \quad Im(z) = 0\}$, where $Re(z)$ is the real part of a complex number $z \in \mathbb{C}$ and $Im(z)$ is the imaginary part of a complex number $z \in \mathbb{C}$. Find a matrix $B \in M_{2\times2}(\mathbb{C})$ such that $W(B) = E$. (You have to check that your B satisfies $W(B) = E$).

3. (8%) Let $(V, <\cdot,\cdot\cdot>)$ be a n-dimensional real inner product space and $\mathcal{B} = \{v_1, v_2, \cdots, v_n\}$ is a basis for V where n is a positive integer. Assume the matrix $A = (a_{ij})$ where

$$a_{ij} = <v_i, v_j>, \text{ for } 1 \le i, j \le n.$$

Show that the matrix A is nonsingular.

4. (10%) Given a matrix $A \in M_{m\times n}(\mathbb{R})$ where m and n are positive integers. The column rank of A is the dimension of the subspace of $\mathbb{R}^m$ spanned by the column vectors of A. The row rank of A is the dimension of the subspace of $\mathbb{R}^n$ spanned by the row vectors of A. Show that the row rank of a matrix A is equal to its column rank.

國立交通大學107學年度碩士班考試入學招生試題

科目：工程數學(4062) 考試日期:107年2月1日 第3節

系所班別：應用數學系數學建模與科學計算碩士班

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Second Part: Ordinary Differential Equations: (*50 points*)

5. (10%) Solve the first order initial value problem

$$\frac{du}{dt} = \cos t + u \cos t, \quad u(0) = 2.$$

6. (15%) Solve the second order initial value problem

$$\frac{d^2u}{dt^2} + 6\frac{du}{dt} + 9u = t^2, \quad u(0) = 1, \quad \frac{du}{dt}(0) = 0.$$

7. (15%) Consider the autonomous equation

$$\frac{du}{dt} = -u^3 - u^2 + u + 1.$$

Determine all the equilibrium solutions and indicate the type of stability of the equilibrium solutions.

8. (10%) Determine the values of λ for which the boundary value problem

$$-\frac{d^2u}{dx^2} = \lambda u, \quad 0 \le x \le 1, \quad \frac{du}{dx}(0) = 0, \quad u(1) = 0,$$

has a nontrivial solution.