

國立交通大學 107 學年度碩士班考試入學招生試題

科目：微積分(4061)

考試日期：107 年 2 月 1 日 第 4 節

系所班別：應用數學系數學建模與科學計算碩士班

第一頁，共一頁

【不可使用計算機】\*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. (10 points) Evaluate  $\int \frac{8x^3 - 12x^2 + 11x - 4}{4x^2 - 4x + 3} dx$ .
2. (10 points) Evaluate  $\int_1^3 (\tan^{-1} x + \cot^{-1} x) \ln x dx$ .
3. (10 points) The government's chief economist announces that the national deficit is increasing, but at a decreasing rate. Interpret this statement in terms of a function and its first and second derivatives.
4. (10 points) Find the volume common to two spheres, each with radius  $r$ , if the center of each sphere lies on the surface of the other sphere.
5. (10 points) Find the area of the surface generated by rotating the parametric curve  $x = \theta - \sin \theta$ ,  $y = 1 - \cos \theta$ ,  $0 \leq \theta \leq 2\pi$ , about the  $x$ -axis.
6. (10 points) Let  $f(x) = x^3 e^{x^5}$ . Find  $f^{(2018)}(0)$ .
7. (10 points) Find an equation for the tangent plane to the surface  $e^z = xyz$  at the point  $(\frac{e}{2}, e, 2)$ .
8. (10 points) Find the arc length of the level curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ .
9. (10 points) Find the area of the region enclosed the polar curve  $r^2 = 4 \cos(2\theta)$ .
10. (10 points) If  $\sum_{n=1}^{\infty} a_n$  is a convergent series, where  $a_n > 0$  for all  $n$ , for what values of  $p$  does  $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n^p}$  always converge. Justify your answer.