【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1.

Let $A=\begin{bmatrix} 1+x_1 & x_2 & x_3 & x_4 & \cdots & x_n \\ x_1 & 1+x_2 & x_3 & x_4 & \cdots & x_n \\ x_1 & x_2 & 1+x_3 & x_4 & \cdots & x_n \\ x_1 & x_2 & x_3 & 1+x_4 & \cdots & x_n \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1 & x_2 & x_3 & x_4 & \cdots & 1+x_n \end{bmatrix}$

Calculate det (A).

(10 points)

2. Apply the Gram-Schmidt algorithm to the following set of vectors:

$$\vec{u_1}=\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}, \quad \vec{u_2}=\begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \quad \vec{u_3}=\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$$

(5 points)

3. Let $A=\begin{bmatrix}10 & -5 & 7\\ -5 & 22 & -5\\ 7 & -5 & 10\end{bmatrix}$

(a) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP=D$.

(5 points)

(b) Applying (a) to find a matrix B such that $B^2=A$.
(You will get full credit if you show how to obtain B. It is unnecessary to carry out the matrix multiplication explicitly.)

(5 points)

4.

(a) By appropriate series expansion, show the following integral can be expanded to a series:

$$\int_0^\infty \frac{(\ln x)^2}{1+x^2}dx = 4\sum_{n=0}^{\infty}(-1)^n(2n+1)^{-3}$$

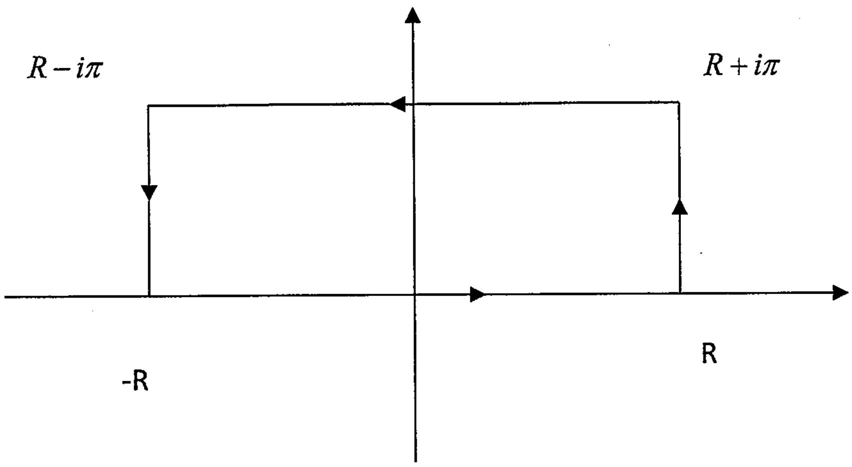
(10 points)

(b) Using contour integration to calculate

$$\int_0^\infty \frac{(\ln x)^2}{1+x^2}dx$$

(15 points)

Hint: $x \to z = e^t$. Try the contour as shown in the figure below, letting $R \to \infty$.



5.

Use Fourier series to find the particular solution of the differential equation with a periodic driving force $m\frac{d^2x}{dt^2} + kx = f(t)$, $f(t) = \pi t, -1 < t < 1$.

(15 points)

6.

Please calculate the unit tangent, unit normal, and unit binormal vectors of the curve $\vec{r}(t) = 3\cos(t)\,\hat{\imath} + 3\sin(t)\,\hat{\jmath} + 4t\hat{k}$. Please calculate the curvature of the curve.

(10 points)

7.

Use power series to find the solutions of the Legendre's differential equation $(1-x^2)y'' - 2xy' + l(l+1)y = 0$.

(15 points)

8.

Solve $x^2y'' - 3xy' + 3y = 2x^4e^x$.

(10 points)