國立交通大學 107 學年度碩士班考試入學試題

科目：工程數學(3101)　　考試日期：107 年 2 月 2 日 第 1 節

系所班別：土木工程學系　　組別：土木系丁組一般生　　第 1 頁，共 2 頁

【可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. (a) For the linear differential equation $y'' + y = 0$, find a solution assuming y(x) in the form of $y(x) = \sin(\lambda x)$. (5%)

   (b) Find another independent solution using the method of reduction of order. (10%)

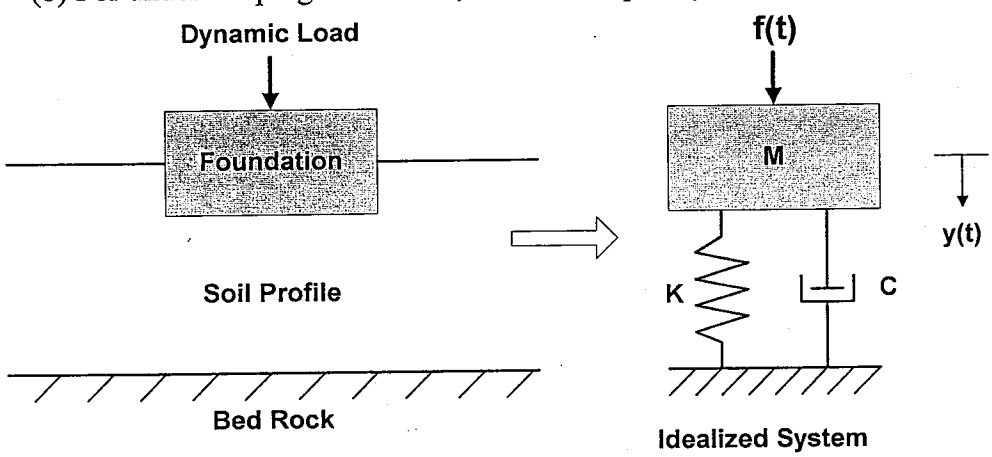
   (c) From the solutions in (a) and (b), what can you say about the general solution for $y'' + y = 0$? (5%)

2. When analyzing the vibration of a foundation subjected to a dynamic loading, it can be idealized as a mass-spring-damper system as shown in the following figure. The equivalent mass, damper, and spring constants are represented by M, C, and K in the governing differential equation

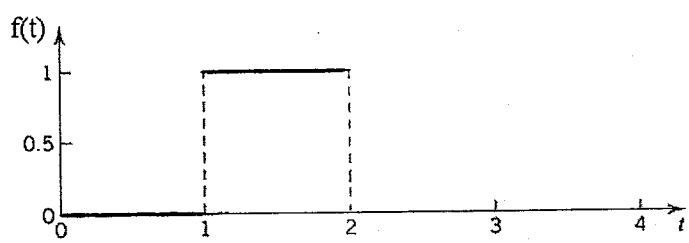
$$My'' + Cy' + Ky = f(t)$$

   (a) Find the condition of critical damping for free vibration (f(t)=0). (5%)

   (b) For underdamping conditions, find the frequency of free vibration. (5%)



3. The above idealized system has an **impulse response** $h(t) = e^{-t}$. Assume the initial conditions $y(0)=y'(0)=0$. Find the response y(t) for the applied force f(t) as shown below. (10%)



Hint: Impulse response h(t) is the response y(t) excited by a unit impulse f(t)=δ(t). In this case, h(t) is the solution y(t) for my''+cy'+ky=δ(t), y(0)=y'(0)=0.

4. One instance of logistic equation is $dy/dt = y - 2y^2$.

   (a) Show (prove, not confirm) that the general solution is $y(t) = \frac{1}{2+ce^{-t}}$, where c is constant. (5%)

   (b) Confirm that the equation y(t)=1/2 is a solution. Is this a particular solution or singular solution?

(5%)

5. Verify that the area of a sphere with radius a is $4\pi a^2$. (10%)

6. Let $f(x,y) = 3$ be the density of mass in the region R: $0 \ll y \ll \sqrt{1-x^2}$, $0 \ll x \ll 1$. Find the center of gravity and the moment of inertia I_x, I_y, I_0. (20%)

7. Find the eigenvalues and eigenvectors of:

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \quad (10\%)$$

8. Find the Fourier transform of e^{-ax^2}, where $a > 0$. (10%)