國立交通大學 107 學年度碩士班考試入學試題

科目：工程數學(3081)　　考試日期:107 年 2 月 2 日 第 1 節

系所班別：土木工程學系　　組別：土木系丙組一般生　　第 1 頁,共 2 頁

【可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. Prove that any matrix can be written in the form of $\mathbf{A}=\mathbf{B}+\mathbf{C}$ if matrix **B** is Hermitian and matrix **C** is skew-Hermitian. (10%)

2. Find the particular solution of $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=4e^{-x}+5\sin x$, where $y=f(x)$ using the method of undetermined coefficients. (15%)

3. Let $f(t)$ be continuous on $0\le t<\infty$, and $f'(t)$ be piecewise continuous on every finite interval contained for $t\ge 0$. If the Laplace transform is denoted as $\mathcal{L}\{f(t)\}=F(s)$ proof the theorem of Laplace transform of a derivative, $\mathcal{L}\{f'(t)\}=sF(s)-f(0)$ (5%).

4. Using the Laplace transformation solve the initial value problem, $y''+3y'+2y=\sin 2t$, where $y(0)=2$ and $y'(0)=-1$ (10%)

5. The governing equation for the problem of 1-D unsteady state heat conduction without convection effect is $\frac{\partial T}{\partial t}=K\frac{\partial^2 T}{\partial z^2}$ where K is a constant value, unit $[K]=\frac{[m^2]}{[t]}$, and $T(z,t)$ is the temperature distribution as a function of depth and time. Find the solution at the initial condition $T(z,t=0)=T_0$ and the boundary conditions of $T(z=0,t)=T_0/2$ and $T(z=\infty,t)=T_0$. (20%)

[**Note:** $\mathcal{L}\left\{1-erf\left[\frac{z}{2\sqrt{t}}\right]\right\}=\frac{1}{s}e^{-\sqrt{s}z}$, $erf(x)=\frac{1}{\sqrt{\pi}}\int_0^x e^{-y^2}dy$]

6. The velocity potential of a flow field is given by $u(x,y)=3xy^2-x^3$. Is the velocity potential a harmonic function? If yes, please find its corresponding conjugate harmonic function. (10%)

7. The velocity of a flow field can be expressed by $\mathbf{u}(x,y,z)=3x^2y^2\mathbf{i}+(2x^3y-e^z)\mathbf{j}+(2z-ye^z)\mathbf{k}$, where $(\mathbf{i},\mathbf{j},\mathbf{k})$ is the unit vector in the direction of each velocity component. Is the flow irrotational? If yes, please find the velocity potential that can indicate the velocity by the definition of $\mathbf{u}=\nabla\phi$? (10%)

8. A cardioid can be expressed by $r = a(1-\cos\theta)$ where $0 \le \theta \le 2\pi$. Find the area of the cardioid. (10%)

9. Proof the identity $\cos^3 x = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$ using the Fourier series expansions. (10%)