

國立交通大學 107 學年度碩士班考試入學試題

科目：工程數學(3051)

考試日期：107 年 2 月 2 日 第 1 節

系所班別：土木工程學系

組別：土木系甲組一般生

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【可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. (15%) Find the exact closed-form general solution for $xy'' + 2y' + 25xy = 0$, where y is a function of x .
2. (15%) Find the recursive relation for the coefficients of the series solution for $\frac{d^4 y}{dx^4} + p(x)\frac{d^2 y}{dx^2} + q(x)y = 0$,

$$\text{where } p(x) = \sum_{n=0,1}^{\infty} a_n x^n, \quad q(x) = \sum_{n=0,1}^{\infty} b_n x^n \text{ and } x \text{ is finite.}$$

3. (20%) Solve the partial differential equation

$$\frac{\partial^4 y}{\partial x^4} + 4\frac{\partial^2 y}{\partial t^2} = \sin 5t, \text{ for } 0 < x < 1 \text{ and } t > 0$$

with zero initial conditions and boundary conditions: $y = \frac{\partial^2 y}{\partial x^2} = 0$ at $x=0$ and 1 .

4. $A = \begin{bmatrix} 2 & 16 & 8 \\ 4 & 14 & 8 \\ -8 & -32 & -18 \end{bmatrix}$

(a) (10%) Find A^{-1} using the method of Gauss-Jordan elimination.

(b) (15%) Find the eigenvalues and eigenvectors of A .

5. (15%) Evaluate the line integral for $\iint_C (1+3xy^2)dx + 3x^2ydy$ using Green's theorem, where C consists of the arc of the parabola $y = 2x^2$ from $(-1,2)$ to $(1,2)$. State the details. Hint: Close C with a horizontal line segment.

6. (10%) Prove $\nabla \times (fV) = \nabla f \times V + f \nabla \times V$

where f and V are scalar and vector functions, respectively, and they are functions of x , y , and z .