

國立清華大學 107 學年度碩士班考試入學試題

系所班組別：資訊工程學系

考試科目（代碼）：基礎計算機科學(2301)

共 4 頁，第 1 頁 *請在【答案卷、卡】作答

1. (12%)
- (a) (6%) How many integer solutions of $x + y + z \leq 15$ satisfy $x \geq 0, y \geq 3, z \geq 5$?
- (b) (6%) In how many ways can we select three distinct integers from 1 to 29 such that their sum is a multiple of 3?
2. (8%) A tree T has $2n$ vertices of degree 1, $3n$ vertices of degree 2, and n vertices of degree 3.
- (a) (6%) Determine the value of n .
- (b) (2%) What is the minimum number of edges that can be added to T so that the resultant graph will have an Euler circuit?
3. (10%) Given a $k \in \mathbb{Z}^+$, the number of derangements d_k is defined as the number of arrangements of $1, 2, \dots, k$ where none of the numbers $1, 2, \dots, k$ are in their natural positions. Take $k = 4$ as an example, $4, 3, 2, 1$ is considered as one of the derangements. We define $d_0 = 1$ for convenience.

For all $n \in \mathbb{Z}^+$, use combinatorial arguments to prove the following equation
$$n! = \binom{n}{0} d_0 + \binom{n}{1} d_1 + \binom{n}{2} d_2 + \dots + \binom{n}{n} d_n = \sum_{k=0}^n \binom{n}{k} d_k.$$

Remarks: No marks will be given if your proof is not a combinatorial one.

4. (10%) On the set \mathbb{Z} , define relation \mathcal{R} by $a\mathcal{R}b$ if $a - b$ is a nonnegative even integer.
- (a) (5%) Prove or disprove that \mathcal{R} is a partial order on \mathbb{Z} .
- (b) (5%) Prove or disprove that \mathcal{R} is a total order on \mathbb{Z} .

Remark: No marks will be given if you do not show your work.

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共 4 頁，第 2 頁 *請在【答案卷、卡】作答

5. (6%) Please answer whether each of the following statements is correct.

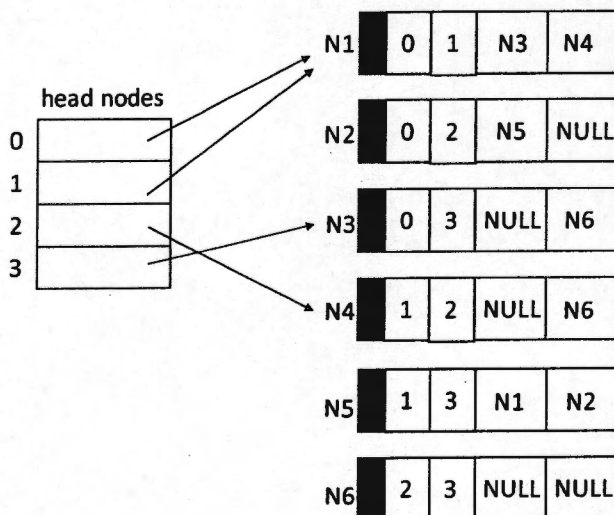
- (a) (1%) $\log n$ is in $O(n^{0.1})$.
- (b) (1%) $n! + n^n$ is in $\Theta(n^n)$.
- (c) (1%) $2^n + n^n$ is in $\Theta(2^n)$.
- (d) (1%) $n!$ is in $\Omega(2^n)$.
- (e) (1%) 2^n is in $O(n^{100})$.
- (f) (1%) $n \log n$ is in $\Omega(n^2)$.

6. (6%)

(a) (3%) Please draw the graph with the following sequential representation.

| | | | |
|--------|--------|--------|-----------|
| [0] 5 | [4] 15 | [8] 2 | [12] 3 |
| [1] 7 | [5] 1 | [9] 3 | [13] 1 |
| [2] 10 | [6] 2 | [10] 0 | [14] 2 |
| [3] 13 | [7] 0 | [11] 1 | [15] NULL |

(b) (3%) Please draw the graph with the following representation of adjacency multilists.



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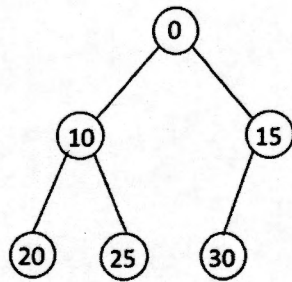
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考試科目（代碼）：基礎計算機科學(2301)

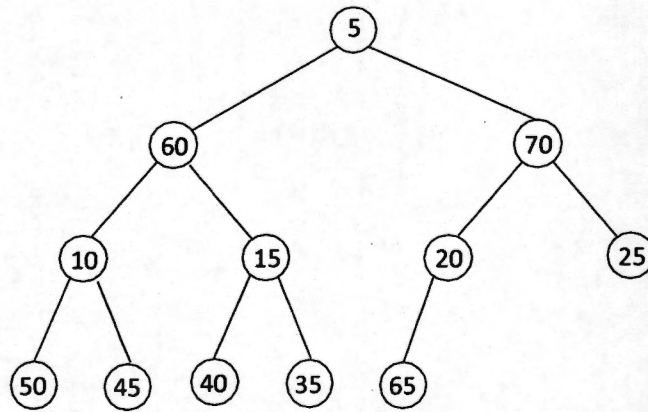
共 4 頁，第 3 頁 *請在【答案卷、卡】作答

7. (18%)

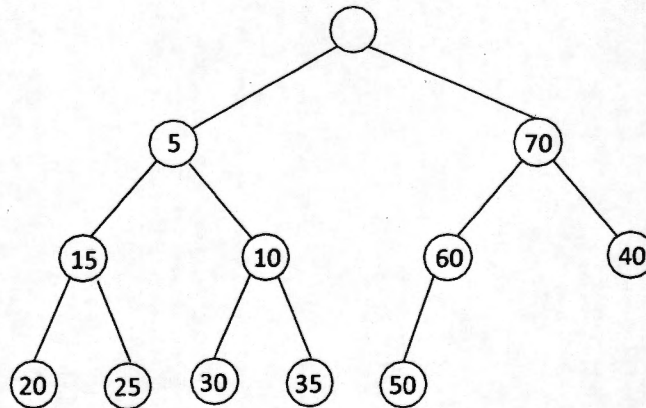
(a) (3%) Please draw the min heap (shown as follows) following a delete min.



(b) (3%) Please draw the min-max heap (shown as follows) following a delete min.



(c) (3%) Please draw the deap (shown as follows) following a delete min.



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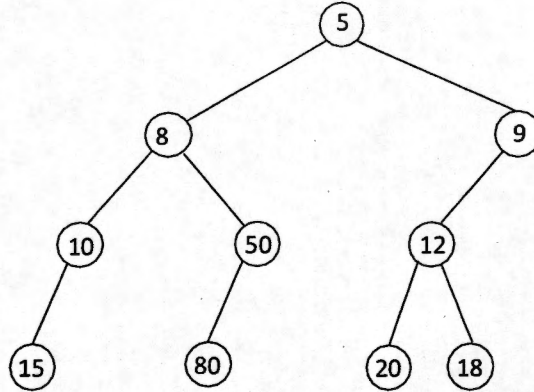
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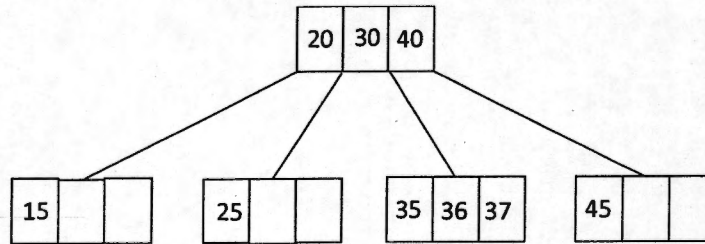
共 4 頁，第 4 頁

*請在【答案卷、卡】作答

- (d) (3%) Please draw the min leftist tree (shown as follows) following a delete min.



- (e) (3%) Please draw the 2-3-4 tree (shown as follows) following the deletion of 15.



- (f) (3%) Please draw the red-black representation of 2-3-4 tree shown in (e).

8. (30%) For an input connected undirected graph G ,

- the HP problem asks if G contains a Hamiltonian path; and
- the HC problem asks if G contains a Hamiltonian circuit.

- (a) (5%) Describe a polynomial-time reduction from HP to HC, and
(10%) explain why such a reduction works.
- (b) (5%) Describe a polynomial-time reduction from HC to HP, and
(10%) explain why such a reduction works.

Remark: No marks may be given if the reduction does not work, or the explanation is incomplete.