題號： 390　　國立臺灣大學 107 學年度碩士班招生考試試題

科目： 應用微積分

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※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。每題 5 分。

1. A parabola is drawn having focus $(0,2)$ and directrix $y=4$. The definite integral representing the arc length of that portion of the parabola on or above the x-axis is given by

(A) $\int_0^{12}\sqrt{4-x^2}dx$

(B) $\int_0^{2\sqrt{3}}\sqrt{4-x^2}dx$

(C) $\int_0^{2\sqrt{3}}\sqrt{4+x^2}dx$

(D) $\int_0^{12}\sqrt{4+x^2}dx$

(E) $\int_0^{12}\frac{dx}{\sqrt{4-x^2}}$

2. Find $\int e^{\frac{2+x}{3}}dx$.

(A) $e^{\frac{2+x}{3}}+C$

(B) $2e^{\frac{2+x}{3}}+C$

(C) $\frac{1}{2}e^{\frac{2+x}{3}}+C$

(D) $\frac{1}{3}e^{\frac{2+x}{3}}+C$

(E) $3e^{\frac{2+x}{3}}+C$

3. Let $f(x)=\frac{x^2+2x-1}{2x^3+1}$, then $f'(1)$ is

(A) -1

(B) 0

(C) 1

(D) 2

(E) None of the above

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4. Let $f(x) = \sqrt{\sec 4x}$, then $f'(x)$ is

(A) $\frac{\tan 4x}{2\sqrt{\sec 4x}}$

(B) $\frac{1}{2\sqrt{\sec 4x}}$

(C) $2\sqrt{\sec 4x}$

(D) $2\sqrt{\sec 4x}\tan 4x$

(E) None of the above

5. Find $\lim_{x\to\sqrt{3}^+} 5^{\frac{1}{3-x^2}}$.

(A) 0

(B) Does not exist and neither ∞ nor $-\infty$

(C) ∞

(D) $-\infty$

(E) None of the above

6. Find the slope of the tangent line at the point $(1,1)$ on the graph of $e^{x-y} = 2x^2 - y^2$.

(A) 0

(B) -1

(C) 1

(D) 2

(E) 3

7. Compute the linearization of $f(x) = \sqrt{x}e^{x-1}$ at $a = 1$.

(A) $L(x) = \frac{3}{2}x + \frac{1}{2}$

(B) $L(x) = -\frac{3}{2}x + \frac{1}{2}$

(C) $L(x) = -\frac{3}{2}x - \frac{1}{2}$

(D) $L(x) = \frac{3}{2}x - \frac{1}{2}$

(E) None of the above

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8. Find $\lim_{x\to 2}\frac{x^3-8}{x^2-4}$.

(A) 0

(B) 1

(C) 2

(D) 3

(E) None of the above

9. If the function $f(x)$ is differentiable and $f(x)=\begin{cases} ax^3-6x & \text{if } x\le 1 \\ bx^2+4 & \text{if } x>1 \end{cases}$, then $a=$

(A) 0

(B) 1

(C) -14

(D) -24

(E) 26

10. Given that the function f is continuous on the interval $[1,\infty)$, and that $\int_1^x \sqrt{f(t)}dt=\sqrt{x}$, then $\int_1^\infty f^2(t)dt=$

(A) 0

(B) $\frac{1}{16}$

(C) $\frac{1}{4}$

(D) 1

(E) ∞

11. Find $\lim_{x\to 1^+}\frac{\exp(x^2-1)}{x-1}$.

(A) 0

(B) 1

(C) 2

(D) ∞

(E) None of the above

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12. $f(x) = (x-1)(x-2)^2(x-3)^3$. Find the value of x that maximizes $f'(x)$.

(A) 0

(B) 1

(C) 2

(D) 3

(E) None of the above

13. $f(x) = x^x$. Find $f'(1)$.

(A) 0

(B) 1

(C) e

(D) $\frac{1}{e}$

(E) None of the above

14. Compute the area of the region enclosed by the graphs of the given equations: $y = e^x$, $y = e^{-x}$, and $x = ln3$.

(A) 1

(B) $\frac{4}{3}$

(C) $\frac{5}{3}$

(D) 2

(E) None of the above

15. $F(x) = \int_0^x (z-3)^2(z-4)^3(z-5)^4 e^{2z} dz$. Find the value of $x \in [0,6]$ that minimizes $F(x)$.

(A) 3

(B) 4

(C) 5

(D) 6

(E) None of the above

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16. Suppose $\lim_{x\to 0^+} f(x) = A$ and $\lim_{x\to 0^-} f(x) = B$. Find $\lim_{x\to 0^-} f(x^2 - x)$.

(A) A

(B) B

(C) $A^2 - B$

(D) $A^2 + B$

(E) None of the above

17. The integral of a constant is a constant.

(A) True

(B) False

18. If $f''(x) = g'(x)$, then $f'(x) = g(x)$.

(A) True

(B) False

19. If $f''(x) > 0$ for all $x \in [a, b]$ and $b > a$, then $\max[f(a), f(b)] > f(z)$ for all $z \in (a, b)$.

(A) True

(B) False

20. If $f''(x) < 0$ for all $x \in [a, b]$ and $b > a$, then there exists $z \in [a, b]$ such that $\max[f(a), f(b)] < f(z)$.

(A) True

(B) False

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