國立政治大學 107 學年度 碩士班 招生考試試題

考試科目	資料結構	系所別	資訊管理學系/科技組	考試時間	2月2日(五)第三節

I. Answer Yes (O)/No (X) for the following descriptions (40%):

1. ______ : The idea of the Divide and Conquer approach is to find the optimum solution by exhaustively searching all possible solutions
2. ______ : The idea of the Dynamic Programming approach is to define global optimum in terms of optimal sub problems. Sub problems may overlap. One can solve and store the results of sub problems bottom-up to accelerate the computation.
3. ______ : In a heap, any value in the left subtree of a node is less than any value in the right subtree of the same node
4. ______ : Removing a key takes O(n) time in a balanced binary search tree with n nodes
5. ______ : An AVL tree is a binary search tree where for every internal node, the heights of its children are the same
6. ______ : In a splay tree, splaying a node means moving the node to the root
7. ______ : While using an unsorted list to implement a map with n nodes, get(k) takes O(1) time
8. ______ : While using a balanced binary search tree to implement a map with n nodes, remove(k) takes O(log n) time
9. ______ : A hash function maps a key to an integer in a fixed interval, e.g., [0,N-1] for a hash table associated with an Array of size N
10. ______ : In a hash table, collision occurs when different keys are mapped to the same index.
11. ______ : Liner probing handles collisions by letting each cell in the table point to a linked list
12. ______ : Separate chaining handles collisions by putting the colliding items in the next available table cell
13. ______ : Hashing is more efficient when the load factor (the number of stored elements / the size of the Array) is higher
14. ______ : The sum of the degrees of all vertices is equal to two times the number of nodes in a graph
15. ______ : The number of edges in a directed graph is less than n(n-1)/2 (n is the number of vertices in the graph)
16. ______ : Checking whether two vertices are adjacent takes O(n) time in an edge-list graph with n vertices
17. ______ : Checking whether an edge is incident to a vertex takes O(n) time in an adjacency-matrix graph with n vertices
18. ______ : Removing a vertex takes $O(n^2)$ time in an adjacency-matrix graph with n vertices
19. ______ : The idea of the Greedy approach is to divide the problem into sub problems that can be solved independently, and conquer the results
20. _______: The minimum spanning tree of a weighted graph must include the edge that has the minimum weight

II. Answer the following questions (60%):

1. A matrix-chain-product is to compute the product of a matrix chain such as $A_0 \times A_1 \times \ldots \times A_{n-1}$, where A_i is a $d_i \times d_{i+1}$ matrix. Consider a simple example $A = A_0 \times A_1 \times A_2$ with $d_0=3$, $d_1=100$, $d_2=5$, and $d_3=5$.

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One can compute the matrix-chain-product in the way $((A_0 \times A_1) \times A_2)$ with 1575 operations. That is, taking 1500 (3*100*5) operations to first compute $A' = A_0 \times A_1$ and then taking 75 (3*5*5) operations to compute the result $A = A' \times A_2$. One can also compute the matrix-chain-product in the way $(A_0 \times (A_1 \times A_2))$ with 4000 operations. That is, taking 2500 (100*5*5) operations to first compute $A' = A_1 \times A_2$ and then taking 1500 (3*100*5) operations to compute the result $A = A_1 \times A'$. As one can see, the order of computation sequence matters. The question is how to find an optimal way to parenthesize the matrixes.

A matrix-chain-product problem is to find an optimal way to compute the product of a given matrix chain with the minimal number of basic (multiplication) operations.

1.1. Use a greedy algorithm.

a. (10%) Describe your strategy.

b. (5%) Show an example that it works.

c. (5%) Show a counterexample that it does not work.

1.2. Use a dynamic programing algorithm. Assume N(i, j) denotes the minimal number of operations to compute the product of $A_i \times \ldots \times A_j$. The optimal solution N(i, j) is to find k in [i, j] such that it is minimal: the sum of two optimal sub problems, N(i, k) and N(k+1, j), plus the number of operations for the last multiplication on the results of the two sub problems.

a. (10%) Show the characterizing equation on N(i,j) with the above optimal relation.

b. (10%) Solve $A = A_0 \times A_1 \times A_2 \times A_3$ with $d_0=3$, $d_1=10$, $d_2=5$, $d_3=5$, and $d_4=6$ with the equation by bottom up computation. Hint: Build a 4 by 4 table for the value of N(i,j). Initially for i = 0..3, N(i, i) = 0. N(0, 3) is the result.

2. Consider the following keys: 19, 39, 28, 5, 36, 18, 25, 26, 33, 53.

a. (10%) Show a hash table that handles collisions with double hashing.
Let N = 17, h(k) = k mod 17. d(k) = 13 – (k mod 13).

b. (10%) Build an AVL tree with the above keys. Insert each key step by step and rebalance the tree when it violates the AVL tree property.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。