

考試科目	微積分	系所別	應用數學系	考試時間	2月3日(星期六)第一節
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1. Determine if each series converges or diverges.

(a) (6%)  $\sum_{n=10}^{\infty} \frac{1}{n \log n \log \log n}$ .      (b) (6%)  $\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$ .

2. Evaluate the limits.

(a) (8%)  $\lim_{x \rightarrow \infty} \left(\frac{e^x + 1}{e^x - 1}\right)^{\ln x}$ .      (b) (8%)  $\lim_{x \rightarrow 4} \frac{\sin^2(\pi x)}{e^{x-4} + 3 - x}$ .

3. Evaluate the integrals.

(a) (8%)  $\int_0^{\frac{\pi}{2}} \frac{1 - \cos x}{1 + \cos x} dx$ .

(b) (8%)  $\int_1^2 \int_{\frac{1}{y}}^y \sqrt{\frac{y}{x}} e^{\sqrt{xy}} dx dy$ .

(c) (8%)  $\int (\ln x)^{\ln x} \left(\frac{1}{x} + \frac{\ln \ln x}{x}\right) dx$ .

(d) (8%)  $\oint_c (6y + x) dx + (y + 2x) dy$ , where  $c: (x - 2)^2 + (y - 3)^4$ .

4. Evaluate the function  $\varphi(t)$  defined by

(a) (10%)  $\varphi(t) = \int_0^{\infty} e^{-\frac{x^2}{2}} \sin xt dx$ .

(b) (10%)  $\varphi(t) = \frac{d}{dt} \int_{\sin^3 t}^{2 + \log_3 t^2} e^{-x^2} dx$ .

5. (10%) Find  $r, s$  and  $t$  such that

$$\lim_{x \rightarrow 0} \left( \frac{\sin 3x \cos 2x}{x^5} + \frac{r}{x^4} + \frac{s}{x^2} + t \right) = 0.$$

6. (10%) Let  $\{a_n\}$  be any positive sequence. Suppose that  $a_{n+1} \leq a_n$  for all  $n \in \mathbb{N}$  and  $\lim_{n \rightarrow \infty} a_n = 0$ . Show that  $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$  is convergent.

備註

一、作答於試題上者，不予計分。  
二、試題請隨卷繳交。