

國立臺灣師範大學 100 學年度碩士班招生考試試題

科目：工程數學(能源應用與車輛技術組)

適用系所：工業教育學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$ (10 分)

2. Solve the general solutions of: (共 15 分)

(a) $y'' + y' - 6y = 0$ (7 分)

(b) $(D+1)^3(D-2)(D+3)y = 0$, where $D \equiv \frac{d}{dx}$ (8 分)

3. Solve $(4x - 10y + 6)dx - (2x + 4y - 6)dy = 0$ (10 分)

4. Prove that the Laplace transform: $L[\cos at] = \frac{s}{s^2 + a^2}$, and $L[\sin at] = \frac{a}{s^2 + a^2}$,

where a is a constant. (10 分)

(Hint: $L[f(t)] = \int_0^{\infty} e^{-st} f(t) dt$, Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$)

5. Derive the inverse Laplace transform: $L^{-1}\left\{\frac{2}{s^2(s+1)^2}\right\}$ (10 分)

6. If a matrix $A = \begin{bmatrix} \cos \omega & \sin \omega \\ -\sin \omega & \cos \omega \end{bmatrix}$, please prove that $A^n = \begin{bmatrix} \cos(n\omega) & \sin(n\omega) \\ -\sin(n\omega) & \cos(n\omega) \end{bmatrix}$.

(10 分)

國立臺灣師範大學 100 學年度碩士班招生考試試題

7. Evaluate the inverse of: (a) $A_1 = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ (5 分) and (b) $A_2 = \begin{bmatrix} 3 & 1 & 0 \\ 2 & 1 & 4 \\ 1 & 3 & 5 \end{bmatrix}$ (5 分)

(共 10 分)

8. A set of dynamic equations for a system can be expressed by: $\begin{cases} x + y' = 0 \\ x' + 9y = 1 \end{cases}$,

$x(0) = y(0) = 0$ (Note: $x' = \frac{dx}{dt}$, $y' = \frac{dy}{dt}$), please solve $x(t)$ and $y(t)$. (15 分)

9. Find all cube roots of -27 (10 分)