

# 國立臺灣師範大學 100 學年度碩士班招生考試試題

科目：工程數學（精密機械組）

適用系所：機電科技學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

## 【試題 1】(10 分)

Given that  $y = \sin x$  is a solution of ODE

$$y^{(4)} + 2y''' + 11y'' + 2y' + 10y = 0,$$

find the general solution.

## 【試題 2】(15 分)

(a) State the convolution theorem for Laplace transform

(b) Find the convolution of  $t * e^t$ ,  $t \geq 0$

(c) Find  $L[t * e^t]$  and with it to verify convolution theorem.

## 【試題 3】(15 分)

For which values of a and b does the following system have

(a) no solution

(b) a unique solution

(c) infinitely many solution.

$$2x + 2y + 3z = 4$$

$$2x + 3y + az = 5$$

$$3x + 4y + 5z = b$$

## 【試題 4】(20 分)

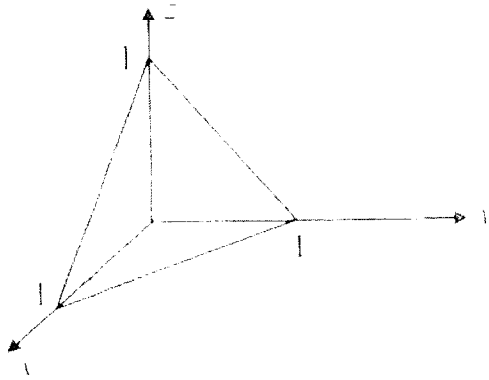
When  $\nabla = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k}$ ,  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  and  $r = \sqrt{x^2 + y^2 + z^2} = |\vec{r}|$ ,

please show that  $\nabla r = \frac{\vec{r}}{r}$ ,  $\nabla \cdot \vec{r} = 3$ ,  $\nabla \times \vec{r} = 0$  and  $\nabla \vec{r} = \vec{ii} + \vec{jj} + \vec{kk}$ .

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【試題 5】(15 分)

$\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ ,  $S$  is the region area bounded with  $x + y + z = 1$ ,  $x = 0$ ,  $y = 0$ ,  $z = 0$ , find  $\oiint_S \vec{F} \cdot \vec{n} dA$ .



【試題 6】(15 分)

Find the Fourier series of the function  $f(x) = \cos^3 2x + \sin^5 x$  using Euler formula.

【試題 7】(10 分)

Evaluate  $\int_C \frac{1}{z} dz$ ,  $C$  is the line segment from  $i$  to  $2 + 4i$