科目：工程數學（光機電系統組）　　　　　　適用系所：機電科技學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. Find a continuous solution satisfying

$$y' + 2y = f(t), \quad where \quad y(0) = 0, f(t) = \begin{cases} 2, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}$$

and plot the output y(t). (17 分)

2. Solve the following differential equation: (16 分)

$$y''' + y'' = e^t \cos(t),$$

3. Solve the general solution of the following system: (17 分)

$$x_1' = 3x_1 + 2x_2 + 2e^{-t};$$
$$x_2' = -2x_1 - x_2 + e^{-t}$$

4. Consider a set of functions $\left\{1, x, x^2 - \dfrac{1}{3}\right\}$ on the interval $[-1, 1]$.

   (a) Is this an orthogonal set? Give a detailed explanation. (5 分)

   (b) Construct an orthonormal set from this set. (5 分)

   (c) Let $g(x) = \alpha_1 + \alpha_2 x + \alpha_3 \left(x^2 - \dfrac{1}{3}\right)$, in which α_1, α_2 and α_3 are constants.

   Define a function $f(x) = \begin{cases} 0, & -1 \le x \le 0, \\ 1, & 0 < x \le 1. \end{cases}$ Please determine the values of

   α_1, α_2 and α_3 so that $\int_{-1}^{1} (f(x) - g(x))^2 dx$ is minimized. (5 分)

5. Let a matrix $\mathbf{A} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$, in which θ is a real number.

   (a) Test the positive definiteness of the matrix. (5 分)

   (b) Find the eigenvalues and eigenvectors of $\mathbf{A}$. (5 分)

   (c) Find the eigenvalues and eigenvectors of $(0.5\mathbf{A})^m$, in which

   $m = 1, 2, 3, \cdots$. (5 分)

6. Let the surface S be that portion of the plane $z = 6 - 3x - 2y$ in the first octant.

   (a) Find a unit normal with a positive $\mathbf{k}$ component. (5 分)

   (b) Let $\mathbf{F}(x, y, z) = z\mathbf{j} + z\mathbf{k}$ represent the flow of a fluid. Please find the flux of

   $\mathbf{F}$ through the surface S <u>oriented upward</u>. (10 分)

7. Find the complex Fourier series of the periodic wave that is the periodic extension

   of the function $f(t) = \cos(t) + \sin(t) + \sin(3t), \quad -\pi < t < \pi$. (5 分)