

國立臺灣師範大學 100 學年度碩士班招生考試試題

科目：工程數學

適用系所：應用電子科技學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. (10 分) Given

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}. \text{ Find } A^{104}, \text{ and } e^{At}.$$

2. (15 分) Find an orthogonal matrix that diagonalizes the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

Check that eigenvectors associated with distinct eigenvalues are orthogonal.

3. (5 分) In C^2 , $\langle x, y \rangle = xAy^*$ is an inner product, where $A = \begin{bmatrix} 1 & i \\ -i & 2 \end{bmatrix}$. Compute

$$\langle x, y \rangle = xAy^* \text{ for } x = (1-i, 2+3i) \text{ and } y = (2+i, 3-2i).$$

4. (10 分) Find the eigenvalues, eigenvectors and Jordan-canonical-form

representation of the matrix $A = \begin{bmatrix} 0 & 4 & 3 \\ 0 & -150 & -120 \\ 0 & 200 & 160 \end{bmatrix}$.

5. Consider the following differential equations:

$$\begin{cases} \dot{x}_1 = 3x_1 + 3x_2 + 8 \\ \dot{x}_2 = x_1 + 5x_2 + 4e^{3t} \end{cases}$$

(10 分) (a) Find the general solutions

(5 分) (b) Find the solutions if the initial conditions are $x_1(0) = 2, x_2(0) = -7$.

6. (10 分) Find the general solution of $y' = \frac{x}{y} + \frac{y}{x}$.

7. (10 分) Solve the initial value problem of $x^2 y'' - xy' = 0; y(2) = 5, y'(2) = 8$.

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8. (10 分) Find the general solution of $y'' - 6y' + 9y = 5e^{3x}$.

9. (10 分) (a) Find 4 solutions of the following differential equation and prove that they are linear independent:

$$\frac{d^4 y}{dx^4} - \frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} - 2y = 0$$

(5 分) (b) What is the general solution of the differential equation?