國立臺灣師範大學 **100** 學年度**碩士班**招生考試試題

科目：**數學基礎** 適用系所：**資訊工程學系**

注意：1.本試題共 3 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. (3 分) A parking lot has 31 visitor spaces, numbered from 0 to 30. Visitors are assigned parking spaces using the hashing function $h(k) = k \textbf{ mod } 31$, where k is the number formed from the first three digits on a visitor's license plate. Which spaces are assigned by the hashing function to cars that have these license plates?
   (1) 317918
   (2) 007100
   (3) 111310

2. (3 分) Find an inverse of 3 modulo 7.

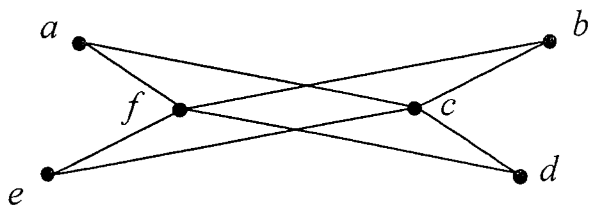
3. (3 分) How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15} to guarantee that at least one pair of these numbers add up to 16?

4. (5 分) Suppose that E, F_1, F_2, and F_3 are events from a sample space S and that F_1, F_2, and F_3 are mutually disjoint and their union is S.
   Find $p(F_2 \mid E)$ if $p(E \mid F_1)=2/7$, $p(E \mid F_2)=3/8$, $p(E \mid F_3)=1/2$, $p(F_1)=1/6$, $p(F_2)=1/2$, and $p(F_3)=1/3$.

5. (6 分) Prove that if n is an integer and $3n+2$ is odd, then n is odd.

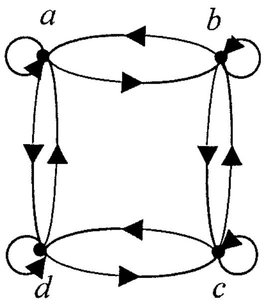
6. (6 分) Find the solution to the recurrence relation
   $$a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$$
   with initial conditions $a_0=1$, $a_1=-2$, and $a_2=-1$.

7. (6 分) Determine whether the graph is bipartite. If it is, write down two sets of vertices that compose the bipartition.



8. (6 分) Determine whether f: $Z \times Z \rightarrow Z$ is onto.
(1) $f(m, n) = |m| - |n|$.
(2) $f(m, n) = m^2 - 4$.

9. (6 分) Determine whether the relation with the directed graph shown as follows is an equivalence relation.



10. (6 分) Suppose that the relation R on a set is represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Is R reflexive, symmetric, and/or antisymmetric?

11. (17 分) Let W be the subspace of R^3 defined by $W = \{ \boldsymbol{x} \mid \boldsymbol{x} = [x_1, x_2, x_3], x_1 + x_2 - 3x_3 = 0\}$. Then, a basis for W is $\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$, where $\boldsymbol{u}_1 = [1\ {-1}\ 0]^t$ and $\boldsymbol{u}_2 = [3\ 0\ 1]^t$.
(a) Apply the Gram-Schmit process to $\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$ to obtain an orthogonal basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$. (5 分)
(b) Calculate the best approximation $\boldsymbol{u}^*$ in W to the vector $\boldsymbol{u} = [1\ {-2}\ 4]^t$. (6 分)
(c) Let $\boldsymbol{v} = [1\ {-2}\ {-4}]^t$. Find $\boldsymbol{w}^*$ in W such that $(\boldsymbol{v} - \boldsymbol{w}^*)^t\boldsymbol{w} = 0$ for all $\boldsymbol{w}$ in W. (6 分)

12. (12 分)
(a) Suppose we have a problem defined as

$$\begin{cases} 20x + 17y + 14z \geq 18(x + y + z) \\ x + 2y + 3z \leq 480 \\ x, y, z \geq 0 \end{cases},$$

in which there are two inequality constraints:

$$20x + 17y + 14z \geq 18(x + y + z) \text{ and } x + 2y + 3z \leq 480,$$

and three simple nonnegativity constraints: $x,\ y,\ z \geq 0$. Transform this problem (hint: by introducing extra (or slack) variables) into an equivalent problem, referred to as an *equality problem* for convenience, that contains only equality constraints and simple nonnegativity constraints. (5 分)

(b) Transform the problem $\begin{cases} 2x+5y \leq 3 \\ -3x+8y \leq -5 \\ x, y \geq 0 \end{cases}$ into an equality problem and find its solution set. (7 分)

13. (10 分) Let A be a given n by n matrix and B be an arbitrary n by n matrix. Let $B^2 = BB,\ B^3 = BBB,\ B^4 = BBBB$, and so forth. If $B^{k+1} = B^k(5I - 3AB^k)$ and $\text{Limit}_{k\to\infty} B^k = M$, where I is the n by n identity matrix and M is nonsingular, what is matrix M?

14. (5 分) If 3 by 3 matrix A has eigenvalues 0, 1, 2, what are the eigenvalues of $A(A-I)(A-2I)$, where I is the 3 by 3 identity matrix.

15. (6 分) Answer "True" or "False" for the following questions:

(a) If matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are identical except in the upper left corner, where $b_{11} = 2a_{11}$, then det(B) = 2det(A).

(b) If A is invertible and B is singular, then $A + B$ is invertible.

(c) If A is invertible and B is singular, then AB is singular.