國立臺灣師範大學 100 學年度碩士班招生考試試題

科目：普通物理　　適用系所：海洋環境科技研究所

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. (共 20 分) A particle with mass m is moving on the x-y plane. Assume that the position of the particle as a function of time t can be written as $\vec{r} = x_0 \sin \omega t \, \hat{i} + y_0 \sin 2\omega t \, \hat{j}$, with x_0, y_0, ω all constants. $\hat{i}$, $\hat{j}$ are unit vectors along the x and y axes respectively.

   A. Calculate the x and y components of the velocity of the particle. (10 分)

   B. Calculate the x and y components of the force acting on the particle as functions of time. (10 分)

2. (共 20 分) A particle with mass m is confined to move on the x axis. A force along the x direction is acting on it and the corresponding potential can be written as: $U(x) = \frac{a}{x^2} - \frac{b}{x}$. Here a and b are positive constants.

   A. Calculate the magnitude $f(x)$ of the force as a function of x. Denote $f(x)$ as positive while the force points in the positive x direction while negative when the force points in the negative x direction. (5 分)

   B. Find the equilibrium point of the force on the positive x axis.(5 分)

   C. If the kinetic energy of the particle when it's very far away ($x \to +\infty$) is $+E_0$ and it is moving toward the origin (原點), what is the closest distance to the origin it could reach (turning point)? E_0 is a positive constant. (5 分)

   D. Describe (numerical details can be omitted) the motion of the particle if its total mechanical energy is negative.(5 分)
   (This is actually the effective potential of an orbiting object, planet or comet, around the sun with x acting as the distance r between the object and the sun.)

3. (共 20 分) Consider a balloon. It is filled with Helium (氦氣) gas on the ground where the air pressure is 1atm and the temperature is 20°C. The volume of the balloon is 5.0 m^3. Assume that the balloon is not tight and hence the pressure of the Helium gas inside is equal to pressure of the air outside. 1atm=1.013×10^5Pa，R=8.31 J/mol K

   A. The helium gas inside is heated on the ground to 50°C. What is the volume of the balloon now? Assume that the balloon is still not tight and the pressure of the Helium gas inside is equal to that of the air outside. (8 分)

   B. Calculate the ratio of the root-mean-square speed (v_{rms}) of the Helium gas molecules inside the balloon before and after the heating.(5 分)

   C. Since the density of the Helium in balloon is smaller than the air outside, the balloon will float and be pushed upward by buoyant force(浮力). Assume the upward move is so fast that the Helium doesn't have time to exchange heat with the air and hence the process can be considered adiabatic (絕熱). When the balloon reaches the height of 6.0 km, the air

pressure is reduced to 0.5atm. Again assume that the balloon is still not tight and the pressure of the Helium gas inside is equal to that of the air outside. What is the temperature of the Helium gas now? (7 分)

4. (共 20 分) Consider a sinusoidal electromagnetic wave. The electric field can be expressed in SI units as:

$$E_z = 10^2 \sin \pi(3\times10^6 x - 9\times10^{14} t)$$

This electric field is in the z direction. Position x and time t are both in SI units.

A. Find the amplitude, frequency and wavelength of this wave. (10 分)

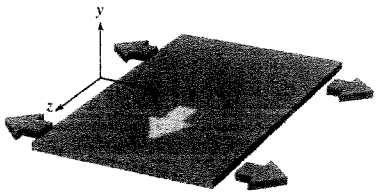
B. Find the direction of the wave propagation. What is the direction of the corresponding magnetic field? What is the maximum magnetic field? (10 分)

5. (共 20 分) The following is a series of problems about the electromagnetic fields. I list the Maxwell Equations here in case you need them:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0},\quad \oint \vec{B}\cdot d\vec{A} = 0,\quad \oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt},\quad \oint \vec{B}\cdot d\vec{s} = \mu_0 i + \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

A. Consider an infinitely large non-conducting plane sheet (無限大平面板) with uniform charge density σ. What is the magnitude of the electric field on either side of the sheet? (8 分)

B. Consider now an infinite conducting plane sheet along the x-z plane carrying a current (traveling in the z direction), as shown in the graph below. The current per unit width (along the x-direction) is i amps per meter.
From symmetry argument we know that the magnetic field above the sheet is along the negative x direction (that is, B_y, B_z both vanish) and in the positive x direction below the sheet. Calculate the magnitude of the magnetic field B. (7 分)



C. Consider a square loop of wire with sides of length 2.0 cm as in the figure. A uniform magnetic field perpendicular to the loop is directed out of page with a magnitude $B = 2.0(\mathrm{T/s^3})\cdot t^3$, where B is in teslas (T) and time t is in seconds. Determine the induced emf (electromotive force) around the square loop at $t = 1.0$ s. (5 分)