1.(25%) A circular bar (rod/shaft) has length $L = 20$ m and cross-sectional radius $R = 1$ m. The bar is fixed at the left end $x = 0$, but free to twist while restrained to elongate at the right end $x = 20$ m. (In other words, the support conditions are fixed-free as a torsional shaft but fixed-fixed as an axial rod.) The bar is simultaneously subjected to a tensile axial load $P = 62.8$ kN and a positive torsional load $T = 31.4$ kN-m, both applied at $x = 10$ m through its cross-sectional centroid. The material is isotropically linearly elastic with Young's modulus $E = 2.6$ GPa and shear modulus $G = 1$ GPa.

(a) Draw the axial force diagram and the torsional moment diagram.

(b) Determine the normal and shear *stresses* and their distributions on the cross section near the left end $x = 0$.

(c) Compute Poisson's ratio; explain its meaning.

(d) Find the maximun principal **strain** and the absolute maximum shear **strain** among those in all locations and inclined planes in the bar.

2.(30%) Suppose a (prismatic, slender) beam is under bending deformation.

(a) Derive the relation between the bending curvature κ and the normal strain ϵ.

(b) Derive the formula for computing the normal stress σ if the moment M is known.

(c) Derive the formula for computing the shear stress τ if the shear force V is known.

(You are encouraged to outline the assumptions made in the derivations and accompany your derivations with due explanations of the quantities, symbols, and/or figures you just used in the derivations and formulae.)

3.(10%) What is the centroid of a cross section? (Please provide one or more examples for illustrations.) How different would it make whether a longitudinal force were applied through the centroid or not?

4.(10%) What is the shear center of a cross section? (Please provide one or more examples for illustrations.) How different would it make whether a transverse force were applied through the shear center or not?

5.(25%) A column with flexural rigidity EI is subjected to a compressive load P acting at the upper end $x = L$. Whenever the load P is at a certain value denoted P_{cr} the column will deflect without limit, *i.e.* buckle, even under a slight disturbance. To describe and model this phenomenon the buckling equation and boundary conditions may be formulated as in the following boundary value problem (BVP):

$$EIw''(x) + Pw(x) = P\delta, \quad 0 < x < L,$$
$$w(0) = 0, \quad w'(0) = 0, \quad w(L) = \delta.$$

(a) What type of support (fixed, pinned, or free) is it at the bottom end $x = 0$?

(b) What type of support is it at the upper end $x = L$?

(c) Solve the BVP for the buckling load P_{cr}.

(d) Solve the BVP for the buckled mode shape $w(x)$.

(e) What is δ? How large is it when $P = P_{cr}$?

(f) If $P < P_{cr}$, what are the values of δ and $w(x)$?

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