(一) Solve the following ODEs (50%; each 10%)

(a) $y'' + 9y = 15e^{x}$

(b) $y''' - y'' + 100y' - 100y = 0$

(c) $y^{iv} - 5y'' + 4y = 10e^{-3x}$; $y'''(0) = y''(0) = y'(0) = y(0) = 0$

(d) $\begin{cases} y_1' = -y_1 - y_2 \\ y_2' = y_1 - y_2 \end{cases}$; $y_1(0) = 0;\ y_2(0) = 1;$

(e) $y'' + 3y' + 2y = \delta(t-1)$; $y(0) = 0; y'(0) = 0$

(二) Non-linear ODE: $y' + xy = xy^{-1}$ (20 %)

(a) Transform the above non-linear ODE into a linear ODE. (8%)

(b) Find the general solution. (8%)

(c) If $y(0) = 3$, Find the particular solution. (4%)

(三) A differential equation for unknown function $y(x)$ (30 %)

$$y'' + 9y' = 0$$

(a) Let $y = e^{\lambda x}$, Find the general solution for $y(x)$. (6%)

(b) Use Laplace Transform Method to find the general solution for $y(x)$. (6%)

(c) Use Power Series Method to find the general solution for $y(x)$.(6%)

(d) Let $y' = p$, reduce to 1st-order ODE and use Separable Variable Method to find the general solution for $y(x)$ (6%)

(e) If $y(0) = 10$, $y'(0) = 9$, Find the particular solution. (6%)