國立中正大學 107 學年度碩士班招生考試試題

系所別：電機工程學系-信號與媒體通訊組
通訊工程學系-通訊甲組
通訊工程學系-通訊丙組

科目：線性代數

第 2 節

1. (10%) How should the coefficients a, b, and c be chosen so that the system

$$ax + by - 3z = -3$$
$$-2x - by + cz = -1$$
$$ax + 3y - cz = -3$$

has the solution x=1, y= -1, and z=2?

2. (10%) A matrix A is said to be skew symmetric if $A^T = -A$. If A is an nxn skew-symmetric matrix and n is odd, show that A must be singular.

3. (15%) Let $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ be a 5×3 matrix. If

$$\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 = \mathbf{a}_2 + \mathbf{a}_3$$

then what can you conclude about the number of solutions of the linear system $A\mathbf{x} = \mathbf{b}$? Explain.

4. (5%) Derive the line in R^3 that contains the point $P(-1,6,0)$ and is orthogonal to the plane $4x - z = 5$.

5. (10%) Are there values of r and s for which $\begin{bmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 3 \end{bmatrix}$ has rank 1? Has rank 2? If so, find those values.

6. There is a set $M = \left\{ \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \middle| m_i \in \{0,1\}, i = 1,2,3, \text{and } 4 \right\}$,
   a. (10%) In M, find all matrices with only one eigenvalue 0.
   b. (10%) In M, find all matrices with two distinct eigenvalues 0 and 1.
   c. (5%) In M, find all diagonalizable matrices with only one eigenvalue 0 (algebraic multiplicity = 2).
   d. (10%) In M, find all orthogonal matrices.
   e. (5%) In M, find all orthonormal matrices.
   f. (10%) Constructing an LU-decomposition for a 2×2 matrix in M with $\prod_{i=1}^{4} m_i = 1$.