

1. [10%] Find the inverse of A by the Gauss-Jordan Method.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 1 & 0 & 0 \\ 1/3 & 1/3 & 1 & 0 \\ 1/2 & 1/2 & 1/2 & 1 \end{bmatrix}$$

2. Consider the following system of equations $Ax = b$

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \quad \text{where } A = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

- (a) [5%] What is the rank of the coefficient matrix A ? Explain your answer.
 (b) [10%] Determine the particular and homogeneous solutions.
3. [15%] A particle moves from the origin $(0,0)$ along the x -axis to $(1,0)$, then along the line segment to $(0,1)$, and then back to the origin $(0,0)$ along the y -axis. Evaluate the work done on this particle by the force field $F(x, y) = x(x+y)\mathbf{i} + xy^2\mathbf{j}$ by using the Green's Theorem.
4. [10%] If A is a unitary matrix, prove that the determinant of A has absolute value 1.
5. [15%] Solve the following boundary value problem:
 $x^2 y'' + xy' - y = \ln(x)$ with $y(1) = 0, y(2) = 0$
6. [15%] By using the method of power series solution only, solve
 $y'' + 4y = 0$ with $y(0) = 1, y(\pi) = 2$
7. [20%] Solve the following two-dimensional Laplace equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\text{with } \frac{\partial u}{\partial x} \Big|_{x=0} = u(0, y), u(\pi, y) = 1, u(x, 0) = 0, u(x, \pi) = 0$$

試題隨卷繳回