

系級	數學系碩士班 B 組(決策科學與海量資料分析)	考試時間	100 分鐘
科目	基礎數學	本科總分	100 分

- (20 points) Find the tangent line of the curve  $\sin(x + y) + \cos(x - y) = y$  at the point  $(x, y) = (-\frac{\pi}{4}, 0)$ .
- (20 points) Find the Taylor formula of the function  $f(x) = \cos(x^2)$  with respect to  $x = 0$  up to the 8-th degree, and prove that  $|\cos(x^2) - (1 - \frac{1}{2}x^4)| \leq \frac{1}{24}$  for all  $x \in [-1, 1]$ .
- (20 points) Graph the curves of functions  $y = x^3 + 2x^2$  and  $y = 3x$  on the same  $xy$ -coordinate. Find the area bounded by the two curves.

- (20 points) Consider the matrix

$$A = \begin{pmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{pmatrix}.$$

Find a basis for the column space, the row space and the null space of  $A$  respectively.

- (20 points) Diagonalize the matrix  $A = \begin{pmatrix} 5 & 1 \\ 0 & 4 \end{pmatrix}$ .

Compute  $A^k$ ,  $k$  is a natural number.