系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1.(20%) Prove the following:

(a) If a sequence $\{\vec{x}_k\}$ in R^n is convergent, then it is Cauchy.

(b) The function $f(x) = \cos x + x$ is uniformly continuous on $\mathbb{R}$.

2.(20%) Suppose that f is three times differentiable on an interval containing a .
   Show that

$$\lim_{h \to 0} \frac{f(a+2h) - 2f(a+h) + f(a)}{h^2} = f''(a)$$

$$\lim_{h \to 0} \frac{f(a+3h) - 3f(a+2h) + 3f(a+h) - f(a)}{h^3} = f'''(a)$$

   Can you find the generalization to higher derivatives?

3. (20%) Let $f(x, y) = \begin{cases} \dfrac{y^2}{x^2 + y^2} & (x, y) \neq (0,0) \\ 0 & (x, y) = (0,0) \end{cases}$.

(a) Prove that f is not continuous at $(0,0)$.

(b) Find $\partial_x f$, $\partial_y f$ and determine whether $\partial_x f(0,0)$ and $\partial_y f(0,0)$ exist or not.

4.(15%) Suppose that f is a homogeneous function of degree a on R^n .

   That is, $f(t\vec{x}) = t^a f(\vec{x})$. Show that $\displaystyle\sum_{j,k=1}^{n} x_j x_k \partial_j \partial_k f = a(a-1)f$

   (f is twice differentiable)

5. (25%) Evaluate the following integrals

(a) $\displaystyle\int_{-1}^{3} \frac{x}{10 - x^2} dx$

(b)Let $S = \{(x, y) : 0 \le y \le 2, y/2 \le x \le 1\}$. Evaluate $\displaystyle\iint_S xy + y^2 dA$

(c) Let $S = \{(x, y) : y \ge 0, x^2 + y^2 \le 1\}$. Evaluate $\displaystyle\iint_S (x + 3y^3) dA$

東吳大學 107 學年度碩士班研究生招生考試試題

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