國立臺灣大學 107 學年度碩士班招生考試試題

1. (20%)
   (a) Prove that every group of order 455 is abelian and cyclic.
   (b) Show that no group of order 56 is simple.

2. (20%)Assume that G is a finite group and p is the smallest prime dividing the order of G.
   (a) Let H be a normal subgroup of order p in G. Show that H is in the center of G.
   (b) Let H be a subgroup of index p in G. Show that H is a normal subgroup of G.

3. (20%)
   (a) Let R be a commutative ring. Show that $R[x]$ is a PID $\Longleftrightarrow$ R is a field.
   (b) Let A_d be the ring of integers in the quadratic field $\mathbb{Q}(\sqrt{d})$. Show that A_5 is a UFD.

4. (16 %)
   (a) Let p be a prime integer and $\mathbb{F}_{p^n}$ be a finite field of order p^n. Show that all subfields of $\mathbb{F}_{p^n}$ are Galois over $\mathbb{F}_p$.
   (b) Show that an algebraically closed field must be infinite.

5. (24 %)
   (a) Determine the Galois group of the splitting field L of $x^5 - 1$ over $\mathbb{Q}$ and the correspondence between subgroups of G and subextensions of L.
   (b) Let $K = \mathbb{Q}(\zeta)$ with ζ a primitive 6th root of unity. Set $f(x) = (x^2 - 2)(x^3 - 2)$ and let L be the splitting field of f over K. Find a such that $L = K(a)$; determine $[L : K]$, and show that L is a cyclic extension of K.

試題隨卷繳回