國立暨南國際大學 107 學年度碩士班入學考試試題

科目:數學(以離散數學、線性代數為主)

2.答案必須寫在答案卷上,否則不予計分。

3.限用藍、黑色筆作答;試題須隨卷繳回。

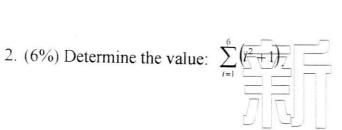
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編號:343

適用: 資工系

(以下各題均須寫出計算或證明過程方予計分)

1. (6%) List all the permutations for the letters a, c, t.



3. (10%) Let $n \in \mathbb{Z}^+$, and $n \ge 14$, prove that n can be written as a sum of 3's and/or 8's.

4. (8%) Let $f, g: \mathbb{Z}^+ \to \mathbb{Z}^+$ where for all $x \in \mathbb{Z}^+$, f(x) = x + 1 and $g(x) = \max\{1, x - 1\}$,

the maximum of 1 and x - 1.



(a) (1%) Is f an onto function?

(b) (1%) Is the function f one-to-one?

(c) (1%) Is g an onto function?

(d) (1%) Is the function g one-to-one?

(e) (2%) Determine $(f \circ g)(x)$ for x = 1, and 7.

(f) (2%) Determine $(g \circ f)(x)$ for x = 1, and 7.

5. (10%) Find the general solution for the recurrence relation

$$a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 3 + 5n, n \ge 0$$

6. (10%) Prove that for every group G, $f(a,b,c) \in G$ and ab = ac, then b = c.

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- 7. (15%) Let $v_1 = (0, 1, 0)$, $v_2 = (-4/5, 0, 3/5)$, and $v_3 = (3/5, 0, 4/5)$.
 - (a) (7%) Show that $S = \{v_1, v_2, v_3\}$ is an orthonormal basis for R^3 with the Euclidean inner product.
 - (b) (8%) Express the vector $\mathbf{u} = (1, 1, 1)$ as a linear combination of the vectors in \mathbf{S} , and find the coordinate $vec\overline{tor}(\overline{u})_S$.
- 8. (5%) Find vectors x any y in R^2 that are orthonormal with respect to the inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 3u_1v_1 + 2u_2v_2$, where $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$, but are not orthonormal with respect to the Euclidean inner product.
- 9. (8%) Find the dimension and a basis for the null space of A, where $A = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 5 & -1 & 1 & -1 \end{bmatrix}.$
- 10. (7%) Prove that if A is a matrix with n columns, then rank(A) + nullity(A) = n.
- 11. (15%) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator given by $T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 x_2 \\ 2x_1 + 4x_2 \end{bmatrix}$.
 - (4%) Find the standard matrix A for T. (a)
 - (6%) Find a basis for R^2 relative to which the matrix for T is diagonal. (b)
 - (c) (5%) Find A^{10} .

