

# 國立臺北大學 107 學年度碩士班一般入學考試試題

系（所）組別：通訊工程學系  
科 目：機率

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☐可 ☒不可使用計算機

1. (5% each, totally 20%) Let  $X$  and  $Y$  be random variables, with the joint PMF:

$$P_{X,Y}(x,y) = \begin{cases} c & \text{if } (x,y) = (1,1), (1,2), (2,1), (2,2); \\ 0 & \text{else,} \end{cases}$$

where  $c$  is a constant.

- (a) What is the constant  $c$  to guarantee that the joint PMF is valid?
  - (b) What is the PMF  $P_X(x)$  of  $X$ ?
  - (c) Let  $W = \min(X,Y)$ . What is the PMF of  $W$ ?
  - (d) What is the CDF of  $W$ ?
2. (5% each, totally 15%) Let  $X$  be the amount of time walking from a park to another. There are two paths between the two parks, denoted by  $H_1$  and  $H_2$ . If path  $H_1$  is chosen, then  $X$  is the exponential random variable, with the mean of 10 minutes. If path  $H_2$  is chosen, then  $X$  is the exponential random variable, with the mean of 20 minutes.
- (a) What is the conditional PDF  $f_{X|H_1}(x)$  given path  $H_1$  is chosen?
  - (b) What is the conditional expectation of  $X$  given path  $H_1$  is chosen?
  - (c) Suppose that a student chooses the two paths *equal-likely*. What is the expected value of  $X$  for the student?
3. (5% each, totally 15%) Let  $X$  and  $Y$  are two *independent* random variables, with the PMFs as follow.

$$P_X(x) = \begin{cases} 1/2 & \text{if } x = 1, 2; \\ 0 & \text{else.} \end{cases}$$

$$P_Y(y) = \begin{cases} 1/2 & \text{if } y = 3, 4; \\ 0 & \text{else.} \end{cases}$$

- (a) What is the moment generating function of  $X$ ?
  - (b) Let  $Z = X + Y$ . What is the moment generating function of  $Z$ ?
  - (c) What is the PMF of  $Z$ ?
4. (24%) Let  $X$  be a random variable uniformly distributed over  $[50,100]$  and  $Y = 200/X$ . Let  $F_X(x)$  and  $F_Y(y)$  denote the CDFs of  $X$  and  $Y$ , respectively, and  $f_X(x)$  and  $f_Y(y)$  denote the PDFs of  $X$  and  $Y$ , respectively. Find  $F_X(x)$ ,  $F_Y(y)$ ,  $f_X(x)$  and  $f_Y(y)$ . (6% each)
5. (26%) Let  $X$  and  $Y$  be jointly distributed random variables with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} k, & 0 \leq x \leq y \leq 2; \\ 0, & \text{otherwise} \end{cases}$$

where  $k$  is a constant.

- (a) Find the value of  $k$ . (2%)
- (b) Find the marginal PDFs  $f_X(x)$ . (6%)
- (c) Find the marginal expectation  $E[X]$ . (6%)
- (d) Find the conditional PDF  $f_{X|Y}(x|y)$ . (6%)
- (e) Find the conditional expectation  $E[X|Y]$ . (6%)

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