國立臺北大學 107 學年度碩士班一般入學考試試題

系（所）組別：金融與合作經營學系

科　　目：統計學

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□可　☑不可使用計算機

一、(10%) Let X_1, X_2, and X_3 be independent and have uniform distribution U(0, 1). The joint p.d.f. is $f(x_1, x_2, x_3) = 1$, where $0 \le x_1, x_2, x_3 \le 1$. What is the probability $P(X_1 + X_2 + X_3 \le 1)$? (Please show the details)

二、(10%) There are two random variables X and Y. Given Y, X follows the conditional distribution $f(x|y) = e^{-y}y^x/x!$, where $x = 0, 1, 2, \ldots$. Moreover, Y follows the distribution $f(y) = \lambda e^{-y\lambda}$, where $y > 0$ and $\lambda > 0$. What are expectation E(X) and the variance Var(X)? (Please show the details)

三、(10%) The moment-generating function of X is $M(t) = (0.25 + 0.75e^t)^{16}$. What are the p.d.f. of X as well as the expectation E(X) and variance Var(X)? (Please show the details)

四、(10%) Let X_1, X_2, ..., X_n be a random sample from $N(\mu, \sigma^2)$ with known mean μ. Moreover, Let $\chi^2_{p(n)}$ denote the $100p^{th}$ percentile of $\chi^2(n)$, the Chi-Square distribution with n degrees of freedom, where $0 \le p \le 1$. Namely, $P(Y \le \chi^2_{p(n)}) = p$, where $Y \sim \chi^2(n)$. Describe how you would construct a 100(1–α)% confidence interval for the unknown variance σ^2, where $0 \le \alpha \le 1$.

五、(10%) Let $f(x;\ \theta) = (1/\theta^2)xe^{-x/\theta}$, where $x > 0$ and $\theta > 0$. Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n for this distribution. What is the maximum likelihood estimator of θ. (Please show the details)

六、X and Y represent the results of throwing two fair dice respectively. Please calculate:

（一）$E(X - Y)$. (3 %)

（二）$Var(X - Y)$. (6 %)

（三）If you have known that X+Y ≤ 3, calculate $Var(X/Y)$. (7 %)

七、X and Y are two random variables, please derive: (12 %)

$Var(X) = E[Var(X|Y)] + Var(E[X|Y])$

八、X~Uniform(0,1) and Y~Uniform(0,1), X and Y are independent. Please derive the probability density functions of the following equations:

（一）$W = X^2$ (10 %)

（二）$W = XY$. (12 %)

試題隨卷繳交