國立臺北大學107學年度碩士班一般入學考試試題

系（所）組別：經濟學系

科　　目：統計學

☑可 ☐不可使用計算機

I、選擇題 （每題 5 分，共 30 分。）

1. Consider the simple linear population model without intercept:

$y_t = \beta x_t + \varepsilon_t$

$\varepsilon_t = \rho \varepsilon_{t-1} + u_t$,

where $u_t \sim i.i.d(0, \sigma^2)$. We use OLS method to estimate parameter $\hat{\beta}$. After we obtain the $\hat{\beta}$, we use OLS residuals to generate Durbin-Watson statistic. Which approximation value does Durbin-Watson statistic converge under $T \to \infty$?

(A) 2- $\hat{\rho}$

(B) $1-2\hat{\rho}$

(C) $2(1-\hat{\rho})$

(D) $1-\hat{\rho}$

(E) $2-3\hat{\rho}$

2. Eric estimates a regression model as below:

$y_t = \alpha + \beta_1 x_t + \beta_2 z_t + u_t$,

Where $u_t \sim N(0, \sigma_u^2)$. If Eric regress x_t on z_t, he find the following equation statistically significant:

$x_t = \omega + \delta_2 z_t + \varepsilon_t$.

Which problem he will face when he use OLS method to estimate $\hat{\beta}_1$ and $\hat{\beta}_2$?

(A) Multicollinearity

(B) Serial correlation

(C) Heteroskedasticity

(D) Nonlinear in parameter

(E) Random sample

3. Mary want to estimates a regression model as below:

$y_t = \alpha + \beta_1 x_{1t} + \beta_2 x_{2t} + \varepsilon_t$,

If she doubts the regression residuals $\hat{\varepsilon}_t$ with heteroskedasticity, she wants to use Breusch-Pagan test to detect whether heteroskedasticity exists in $\hat{\varepsilon}_t^2$. Which independent variables set she will choose to execute Breusch-Pagan test?

(A) $x_{1t} x_{2t}$

(B) x_{it}, x_{2t}, x_{1t}^2, x_{2t}^2

(C) x_{it}, x_{2t}, x_{1t}^2, x_{2t}^2, $x_{1t} x_{2t}$

(D) x_{it}, x_{2t}

(E) x_{it}, x_{2t}, $x_{1t} x_{2t}$

4. Consider the simple regression

$y_t = \alpha + \beta x_t + \varepsilon_t$

And suppose population parameter $\beta = 2$ and least square estimator $\hat{\beta} = 2.3$. Now consider the regression

$(x_t - y_t) = \alpha^* + \beta^* x_t + u_t$.

What is the value of $\hat{\beta}^*$?

(A) 1

(B) -1

(C) -1.3

(D) -0.3

(E) 0.3

5. Consider the linear model with intercept:

$y_t = \alpha + \beta x_t + u_t$

If the $A = \sum_{t=1}^{T}(x_t - \bar{x})^2$ is positive definite and $E(u_t \mid x_t) = 0, \forall t = 1,2,\ldots,T$, which statement is true under sample size $T \to \infty$.

(A) $\hat{\beta}$ is consistent, but we cannot judge whether $\hat{\beta}$ is efficiency

(B) $\hat{\beta}$ is unbiased and efficiency

(C) $\hat{\beta}$ is consistent and efficiency

(D) $\hat{\beta}$ is, but we cannot judge whether $\hat{\beta}$ is efficiency

(E) $\hat{\beta}$ is unbiased and consistent.

6. Consider the linear population model with intercept:

$y_t = \alpha + \beta x_t + \varepsilon_t$

The variance structure is known as below:

$E(\varepsilon_t^2) = \sigma^2 h_t^2$,

Which regression may obtain efficiency $\hat{\beta}$ estimator?

(A) Regress y_t on 1, x_t

(B) Regress y_t/h_t on $1/h_t$, x_t/h_t

(C) Regress y_t/h_t^2 on $1/h_t^2$, x_t/h_t^2

(D) Regress y_t on $1/h_t$, x_t/h_t

(E) Regress y_t/h_t on 1, x_t

II、計算問答題 （每大題 10 分，共 20 分。）

1. Mary has $T = 40$ observations on variables y, x_1, and x_2. She estimates the following equations by least squares:

$$y = -2.0 + 3.5x_1 - 1.5x_2, \qquad (1)$$
$$(0.7) \quad (0.3)$$

Residual sum of squares (SSR)=140, $R^2 = 0.8$ where the numbers in parentheses are standard errors. What is the true value of SST in equation (1)? What is the value of s^2 (estimated error variance.)?

2. Consider the simple regression model, assume to satisfy ideal conditions:

$y_t = \beta x_t + \varepsilon_t \qquad (2)$

We now want to consider another estimator $\tilde{\beta}$, defined as follow. You pick two observations at random; say, $t = 3$ and $t = 6$.

$\tilde{\beta} = (y_6 - y_3)/(x_6 - x_3) \qquad (3)$

(a). Consider the estimator $\tilde{\beta}$. Whether is $\tilde{\beta}$ unbiased?

(b). Compared with OLS estimator $\hat{\beta}$, which estimators is efficiency? Please explain.

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Ⅲ、填空題（每格5分，共50分。）

1. Let random variable X follow a geometric distribution with parameter p, that is,

$$P(X=x)=pq^{x-1},\quad p+q=1,\quad x=1,2,3,...$$

Calculate the probability of $P(X>m)$, where m is a positive integer. _____(1)_____

2. Suppose $S_n=X_1+X_2+\cdots+X_n$, where $E(X_i)=\frac{1}{n}$, $Var(X_i)=\frac{1}{n}-\frac{1}{n^2}$ and $Cov(X_i,\ X_j)=\ \frac{1}{n(n-1)}-\frac{1}{n^2}$ *for* $i\neq j$.

Then, $Var(S_n)=$_____(2)_____.

3. Suppose that the probability density function of X is $f(x)=2x\exp(-x^2)$, $0\le x<\infty$. Find the probability density function of $Y=X^2$, $f(y)=$_____(3)_____.

4. Suppose that the continuous random variables (X,Y,Z) have a valid joint probability density function

$$f(x,y,z)=Kxyz^2,\quad 0\le x,y\le 1,\quad 0\le z\le 3.$$

Then, $E(Y)=$_____(4)_____ and $E\left(Y\middle|X=\frac{1}{2},Z=1\right)=$_____(5)_____. Of course, you should find the value of K first.

5. If random variables $X_1,X_2,...,X_n$ are i.i.d. $N(\mu,\sigma^2)$, and S^2 is their sample variance. Then, the distribution of $\frac{(n-1)S^2}{\sigma^2}$ is _____(6)_____.

6. Suppose you play a game of chance where you win w dollars with probability p but lose v dollars with probability $1-p$. Denote X_i the random variable. Then, how large should p be so that you, a reasonable person, are willing to play the game? _____(7)_____

7. Let $X_1,...,X_n,X_{n+1}$ be a random sample from a normal population having an unknown mean μ and variance 1, and $\bar{X}_n$ be the average of the first n variables. Then, the distribution of $X_{n+1}-\bar{X}_n$ is _____(8)_____. If $\bar{X}_{200}=5$, then the 95% confidence interval for X_{201} is _____(9)_____.

8. In a multiple linear regression, a dependent variable is explained by a constant and 4 independent variables using 35 observations. The R^2 of this regression is 0.6. To test the significance of this regression, we can obtain the F statistic equal to _____(10)_____.

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