

國立臺灣科技大學 107 學年度碩士班招生試題

系所組別：化學工程系碩士班
 科目：工程數學與輸送現象

總分 100 分，請依序作答，並詳列計算過程。

1. Solve the Bernoulli differential equation: $xy' + y = 3x^2y^4$ with IC: $y(1)=0.5$. (10%)
2. Solve the initial value problem: $y'' + 9y = f(t)$; $y(0) = 0, y'(0) = 1$, where $f(t) = 0$ for $t < 2$ and $f(t) = t$ for $t \geq 2$. (15%)
3. Solve the boundary value problem using the method of separation of variables:

$$\nabla^2 u(x, y, z) = 0 \quad \text{for } 0 < x < 1, 0 < y < 2, 0 < z < 4,$$

$$u(0, y, z) = u(1, y, z) = 0 \quad \text{for } 0 < y < 2, 0 < z < 4,$$

$$u(x, 0, z) = u(x, 2, z) = 0 \quad \text{for } 0 < x < 1, 0 < z < 4,$$

$$u(x, y, 0) = 0, \quad u(x, y, 4) = f(x, y) = xy \quad \text{for } 0 < x < 1, 0 < y < 2$$
 (17%)
4. Write the function value in the form $a + bi$ for $\sqrt[3]{2i}$. (8%)
5. Laminar flow of water goes through a tube of inner diameter 0.25 cm, the maximum velocity (u_{\max}) is 1.2 cm s^{-1} . Calculate the shear stress on the tube wall. The water viscosity at room temperature is $0.01 \text{ poise (g cm}^{-1} \text{ s}^{-1})$, and the velocity profile of laminar flow is parabolic; expressed as $u_x = u_{\max}[1 - (r/R)^2]$, with R : tube radius; r : radial position; u_x : water velocity at position r . (10%)
6. One way to separate the uranium isotope (鈾同位素) is to take advantage of the mass difference between U^{235}F_6 and U^{238}F_6 which diffuse through a porous membrane with different Knudsen diffusivity (D_{KA}) at elevated temperatures. Please estimate the ratio of Knudsen diffusivity $D_{KA-\text{U}^{235}\text{F}_6}/D_{KA-\text{U}^{238}\text{F}_6}$. (10%)
 Molecular weight of U^{235}F_6 is 349.028, that of U^{238}F_6 is 352.038, and

$$D_{KA} = 97.0 \bar{r} \sqrt{\frac{T}{M_A}} \quad T: \text{temperature in K; } M_A: \text{molecular weight of species A; } \bar{r}: \text{pore radius in m.}$$
7. Select the most appropriate answer to the following questions (one choice only), explain your choice. Engineers have designed a short flow manifold, as shown in Figure 1, to achieve certain effects on the water flow. The flow manifold is featured with a large pipe (diameter 8 in.) filled with many straws (5-6 mm in diameter, 15-20 cm in length). We connect the manifold at the end of pipe and allow water flow through; (5%)
 - (A) The flow type changes from laminar into turbulent flow, the pressure drop decreases.
 - (B) The flow type changes from turbulent into laminar flow, the pressure drop decreases.
 - (C) The flow type changes from laminar into turbulent flow, the pressure drop increases.
 - (D) The flow type changes from turbulent into laminar flow, the pressure drop increases.

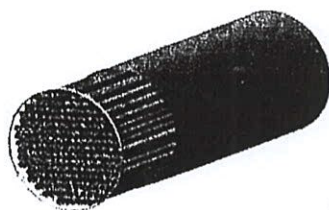


Figure 1. Laminar flow manifold



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8. A heat exchanger of one shell pass and two tube passes is designed to warm up the cold water flow 12.6 kg s^{-1} from 15.5 to 46.5 °C, while the hot water enters at 98.5 °C and leaves at 49.0 °C, calculate (A) ΔT_{lm} and ΔT_m ; (B) the required inside heat transfer area of the heat exchanger (m^2). The specific heat of water is $4180 \text{ J kg}^{-1} \text{ K}^{-1}$, overall heat-transfer coefficient based on inside surface area U_i is $3500 \text{ W m}^{-2} \text{ K}^{-1}$ (A: 10% ; B: 5%) (15%)

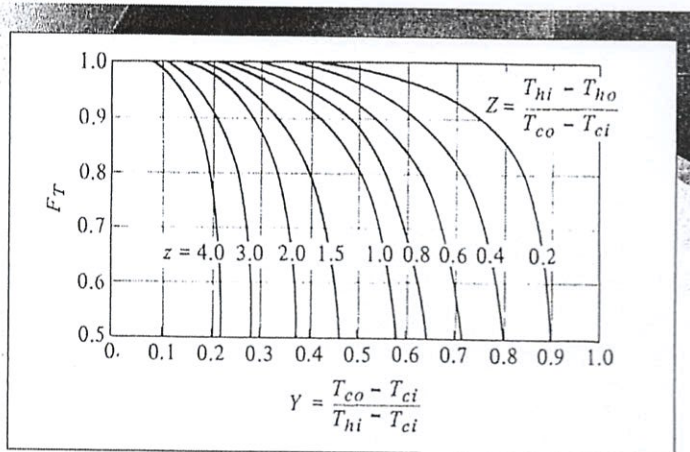


Figure 4.9-4(a) Correction factor to LMTD for 1-2 and 1-4 exchangers. (Geankoplis, 4th ed.)

Figure 2 Correction factor F_T .

$$q = U_i A_i \Delta T_m$$

$$\Delta T_m = F_T \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln[(T_{hi} - T_{co}) / (T_{ho} - T_{ci})]}$$

9. A binary mixture of A and B, 100 kg mol in total, containing 55 mol% A and 45 mol% B, undergoes equilibrium (or flash) distillation, and 58 kg mol is distilled. Estimate the molar fraction of A in the vapor (y_A) and in the liquid (x_A). Relative volatility (α_{AB}) is 2.86.

$$\alpha_{AB} = \frac{y_A / x_A}{y_B / x_B} = \frac{y_A / x_A}{(1 - y_A) / (1 - x_A)} \quad (10\%)$$

