國立臺灣科技大學 107 學年度碩士班招生試題

系所組別：自動化及控制研究所碩士班

科　　目：工程數學

(總分為 100 分)

1. (a) Use Laplace transform to solve the initial value problem as follows:

$$\begin{cases} x' + 2y' - x = 0, \\ 4x' + 3y' + y = -6, \quad x(0) = y(0) = 0. \end{cases}$$

(5%)

(b) Laplace transform: $L\{f(t)\} = F(s)$ and $L\{g(t)\} = G(s)$.

Prove the convolution theorem $L\{f(t) * g(t)\} = F(s) \cdot G(s)$.

(5%)

2. A function f is called *analytic* at x_0 if $f(x)$ has a power series representation in some interval $(x_0 - h,\ x_0 + h)$ about x_0. Find a power series solution of $y'' - xy' + y = 3$ expanded about $x_0 = 0$, and use it to generate the first five non-zero terms. (10%)

3. Find the solutions of the following differential equations.

(a) $y'' + 4y' = 8 + 34\cos(x), y(0) = 3, y'(0) = 2.$ (8%)

(b) $2y^2 + ye^{xy} + (4xy + xe^{xy} + 2y)y' = 0.$ (7%)

4. Apply the matrix operation (**Ax=B**) to calculate the least squares line $y=ax+b$ for the following data (x_i, y_i)= (-3,-23), (0,-8.2), (1,-4.6), (2,-0.5), (4, 7.3),(7, 19.2)

(15%)

5. Consider the following two vectors **F** and **G**.

F= $-3i+6j+k$,　**G**= $-i-2j+k$

(a) Compute **F•G** and the angle in degrees between those two vectors. (5%)

(b) Compute **F×G** , **G×F** and verify the anticommutativity of the cross product.

(5%)

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6. Consider a 3x3 matrix $A=\begin{bmatrix} 3 & 0 & 0 \\ 1 & -2 & -8 \\ 0 & -5 & 1 \end{bmatrix}$

(a) Find the eigenvalues and the corresponding eigenvectors. (6%)

(b) Sketch the Gershgorin circles (center and radius) and locate the eigenvalues as points in the plane. (4%)

7. Let $f(x)$ be defined on $[-L, L]$, the Fourier series of $f(x)$ is expressed as

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}[a_n \cos(n\pi x/L) + b_n \sin(n\pi x/L)]$$

Solve a_0, a_n and b_n.

(10%)

8. Let $H(t)$ be the Heaviside function, defined by

$$H(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0. \end{cases}$$

Calculate the Fourier transform of $f(t) = H(t)e^{-5t}$.

(10%)

9. (a) Let $z = 1 + i$, solve r and θ for its polar form $z = re^{i\theta}$. (5%)

(b) Evaluate $\int_\gamma f(z)\,dz$ for $f(z)=|z|^2$; γ is the line segment from $-i$ to 1. (5%)

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