國立臺灣師範大學107學年度碩士班招生考試試題

科目：**機率與統計**　　　　　　　　適用系所：**數學系**

注意:1.本試題共1頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。3.答案必須有計算過程，否則會斟酌扣分。

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1. (20分) Let $X_1, \cdots, X_n$ be independent and identically distributed Uniform$(0,1)$ random variables. We say that a **record** occurs at position i, $i \le n$, if $X_j \le X_i$ for all $1 \le j \le i$.

   (a) Let $R_i = 1$ if a record occurs at position i and 0 otherwise. Find the probability distribution of R_i.

   (b) Find the mean and variance of the number of records.

2. (20分) Let $Y = e^X$ where X is normally distributed with mean μ and variance σ^2.

   (a) Find the probability density function of Y.

   (b) Find the expectation of Y.

   (c) Find the variance of Y.

   (d) Find the mode of the distribution of Y.

3. (20分) Consider the joint distribution of two random variables (X, Y). Suppose that the density function of X is $f(x) = \theta e^{-\theta x}$, $x \ge 0, \theta > 0$, and given $X = x$, the conditional distribution of Y is $f(y|x) = e^{-\beta x}(\beta x)^y / y!$, $y = 0, 1, \cdots, x \ge 0, \beta > 0$.

   (a) Find the marginal probability density function of Y.

   (b) Find E(Y), Var(Y) and Cov(X, Y).

4. (20分) Let X be a random variable having probability density function $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$. To test $H_0 : \theta \le 1$ against $H_1 : \theta > 1$, the critical region $C = \{x : \frac{9}{10} \le x\}$ was used.

   (a) Find the power function of the test.

   (b) Find the size of the test.

5. (20分) For a simple linear regression model with no intercept: $Y_i = \beta x_i + \varepsilon_i$ where ε_i for $i = 1, \cdots, n$ are independent and $\mathcal{N}(0, \sigma^2)$.

   (a) Find the maximum likelihood estimators of β and σ^2.

   (b) Assume that β and σ^2 are both unknown. Derive the likelihood ratio test for $H_0 : \beta = 0$ against $H_1 : \beta \neq 0$.

(試題結束)