國立臺南大學 107 學年度 **資訊工程學系碩士班** 招生考試 離散數學與線性代數 試題卷

一、離散數學

1. How many permutations of the letters of alphabet (10%)

   A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, R, S, T

   contain neither the pattern "CAT" nor the pattern "DOG"?

2. Negate and simplify the following statement (10%)

   $$\forall x \exists y \exists z[(p(x) \vee q(x)) \rightarrow r(y,z)]$$

3. For $n \geq 0$ let F_n denote the nth Fibonacci number. Use mathematical induction to prove that (10%)

   $$F_0 + F_1 + F_2 + \cdots + F_n = \sum_{i=0}^{n} F_i = F_{n+2} - 1.$$

4. For each of the following function f: $\mathbf{R} \rightarrow \mathbf{R}$, verify whether $y = f(x)$ is invertible? Find the inverse function $f^{-1}(y)$ for $f(x)$ if it exists.

   (a) $f = \{(x, y) | 4x + 9y = 11\}$ (5%)

   (b) $f = \{(x, y) | x^2 = y - 1\}$ (5%)

5. Let $A = \{1, 2, 3, 4\}$, give a relation $\mathcal{R}$ on the set A ($\mathcal{R} \subseteq A \times A$) satisfying the following properties:

   (a) A relation $\mathcal{R}_1$ has symmetric and antisymmetic at the same time. (5%)

   (b) A relation $\mathcal{R}_2$ has reflexive, antisymmetic, and transitive containing the elements(1, 2) and (2, 4). (5%)

二、線性代數

1. Find the determinant of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 2 & 1 & 1 & 2 & 2 \\ 0 & 3 & 0 & 0 & 3 & 2 \\ 0 & 4 & 0 & 0 & 0 & 1 \end{bmatrix}$. (10%)

2. Find the least squares solution of the following linear system: (10%)

$$x_1 - x_2 = 1$$
$$2x_1 + x_2 = -1$$
$$x_1 + x_2 = -2$$

3.Find a basis for the row space of $A = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$, which consists entirely of row vectors from A. (10%)

4.Determine the kernel and the range of the following linear transformation L: $R^2 \to R^3$: $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ 0 \\ 2x \end{bmatrix}$, for each vector $\begin{bmatrix} x \\ y \end{bmatrix} \in R^2$. (10%)

5.Let W be the subset of R^3 consisting of all vectors of the form $(a, -a, 0)$. Show that W is a subspace of R^3. (10%)