國立臺南大學107學年度 **應用數學系碩士班** 招生考試　微積分　試題卷

1. Find the indicated limits. (15%)

   (1) $\lim_{x\to-\infty} x\left(\sqrt{9x^2-2}+3x\right)=?$

   (2) $\lim_{x\to0^+} x^{\sin x}=?$

   (3) $\lim_{x\to\infty}\left|\frac{x+1}{x-2}\right|^{\sqrt{x^2-4}}=?$

2. Let f be a differentiable function, and $g(x)=f\left(\frac{x}{f(x)}\right)$. Compute the derivative of $g(x)$. Express your answer in terms of $f(x)$ and $f'(x)$. (10%)

3. Find the equation of the line that tangents to the curve of $x^2y^2-2x=4-4y$ at the point $(2,-2)$. (15%)

4. Compute the indefinite integral $\int \frac{1}{x+\sqrt{x}}\,dx=?$ (15%)

5. Let $f(x)=\frac{1}{1+e^{-x}}$. Show that $f'(x)=f(x)(1-f(x))$. (15%)

6. Prove or disprove: if $f'(x)$ exists in $(-1,1)$ and $f'(0)>0$, then there is a small interval $(-\varepsilon,\varepsilon)$ around 0 such that $f'(x)>0$ in this interval. (15%)

7. Let $f(x)=x^3-3x^2-9x+2$. Find the absolute maximum and minimum values on the interval $[-5,5]$. (15%)