1. (10%) Consider the following initial value problem

$$\frac{dy}{dx} - y^2 = 0;\ y(0) = 1.$$

Find the equation of the tangent line that passes $(x, y) = (\frac{1}{2}, 2)$ on the solution curve.

2. (10%) Evaluate

(a) (5%)

$$\mathscr{L}^{-1}\left\{\frac{(2s+5)e^{-2s}}{(s-3)^2}\right\}$$

(b) (5%)

$$\mathscr{L}\left\{\int_0^t e^{2\tau}\cos(3(t-\tau))d\tau\right\}$$

3. (10%) Solve the following initial value problem:

$$\begin{aligned}\frac{dx}{dt} &= 2x + 3y,\\ \frac{dy}{dt} &= 2x + y,\\ x(0) = 4,\ y(0) &= 1.\end{aligned}$$

4. (10%) Find the value of k such that the given differential equation is exact.

$$(6xy^3 + \cos y)dx + (2kx^2y^2 - x\sin y)dy = 0$$

5. (10%) Find the following initial value problem

$$x\frac{dy}{dx} + y = x^2y^2,\ y(1) = 2.$$

6. (10%) Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 0, & x < -1 \\ -1, & -1 < x < 0 \\ 2, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$$

7. (10%) Find product solutions of the differential equation

$$\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 0$$

8. (15%) Find the eigenvalues and eigenfunctions of the boundary-value problem

$$y'' + y' + \lambda y = 0, \quad y(0) = 0, \quad y(2) = 0.$$

9. (15%) Solve the initial-value problem

$$y'' + x(y')^2 = 0, \quad y(1) = 4, \quad y'(1) = 2$$