

國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：統計學【經濟所碩士班】

題號：403003

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 1 頁第 1 頁

Answer the following five questions, equally weighted

請務必依題序在答案卷上作答 (5 大題, 共 100 分)

1. (20%) A random sample of size $n = 1$ is drawn from a uniform pdf defined over the interval $[0, \theta]$, i.e.

$$f_Y(y; \theta) = \begin{cases} \frac{1}{\theta}, & 0 < y < \theta; \\ 0, & \text{otherwise.} \end{cases}$$

We decide to test $H_0 : \theta = 2$ versus $H_1 : \theta \neq 2$ by rejecting H_0 if either $y \leq 0.1$ or $y \geq 1.9$, where y is the value drawn. Find the type I error α . Also, find the type II error β if the true value of θ is 2.5. ■

2. (20%) Suppose X is a random variable with distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0; \\ 0.4 & \text{if } 0 \leq x < 1; \\ 1 & \text{if } x \geq 1. \end{cases}$$

Find the density function of X , $f(x)$. ■

3. (20%) Let (X, Y) be a continuous random vector with joint density function $f(x, y) = cx^2y, 0 < x < 1, 0 < y < 1$.

(a) What is c ?

(b) Are the events $\{X \leq 0.5\}$ and $\{Y \leq 0.5\}$ independent? ■

4. (20%) Let X_1, \dots, X_n be independently identically distributed with X_i having density $f(x; \theta) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$. Find the MLE of θ . ■

5. (20%) Let Y_1, Y_2, \dots, Y_n be a random sample from the Bernoulli pdf,

$$f_{Y_i}(y_i; p) = p^{y_i}(1-p)^{1-y_i}, \quad y_i = 0, 1; \quad 0 < p < 1.$$

It has already been suggested that

$$W = \frac{Y}{n}$$

be used as an unbiased estimator for p , where $Y = \sum_{i=1}^n Y_i$. How does $\text{Var}(W)$ compare with the Cramer-Rao lower bound? ■