

國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【海工系碩士班甲組】

題號：459001

※本科目依簡章規定「不可以」使用計算機(問答申論題)

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1. 【Ordinary Differential Equations】 (20%)

(a) Solve $(x^2 D^2 - 4xD + 6I)y = 21x^{-4}$ (10%)

(b) Solve $y'' + y = \cos x - \sin x$ by undetermined coefficients and variation of parameters respectively. (10%)

2. 【Vector Calculus】 (20%)

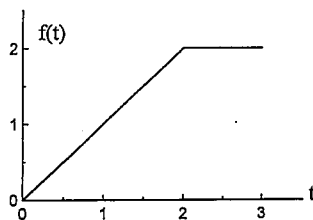
Evaluate $\iint_S (\text{curl} \vec{F}) \cdot \vec{n} \, dA$, $\vec{F} = [z^2, \frac{3}{2}x, 0]$, $S: 0 \leq x \leq a, 0 \leq y \leq a, z = 1$.

(a) directly for given \vec{F} and S (10%); (b) verify "Stokes's Theorem" (10%).

3. 【Laplace Transform】 (20%)

(a) Find the Laplace transform. (10%)

$$f(x) = \begin{cases} t & 0 < t < 2 \\ 2, & 2 < t < 3 \\ 0 & t > 3 \end{cases}$$



(b) Find the inverse transform by convolution and differentiation respectively.. (10%)

$$\frac{2\omega^2}{(s^2 + \omega^2)^2}$$

4. 【Fourier Analysis】 (15%)

A function $f(x) = 2 + x^2$, $0 \leq x \leq 2$ is expanded into :

(a) Fourier sine series (5%); (b) Fourier cosine series (5%); (c) Fourier series (5%)

For $x = 2$, what values of each series converge to.

5. 【Partial Differential Equation】 (15%)

Solve the following PDE using the Method of Separation of Variables.

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial x^2}, \quad 0 < x < L, \quad t > 0$$

boundary conditions : $\phi(0,t) = \phi(L,t) = 0, \quad t > 0$

initial conditions : $\phi(x,0) = x(L-x), \quad \phi_t(x,0) = 0$

6. 【Complex Analysis】 (10%)

Evaluate $\int_0^{2\pi} \frac{d\theta}{3 - \cos \theta}$