

# 國立中山大學 107 學年度碩士暨碩士專班招生考試試題

科目名稱：微積分【企管系企管甲班碩士班甲組選考、乙組選考、丙組選考】 題號：441003

※本科目依簡章規定「不可以」使用計算機(問答申論題)

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請按題號順序於答案卷作答，並寫出計算過程

1. Differentiate the following functions (25%)

i.  $f(t) = (t + \frac{1}{t^3})^2$  by chain rule (5%)    ii.  $f(t) = (\cos 3t)^2$  (5%)

iii.  $f(x) = \ln \frac{\sin x}{x}$  (5%)    iv. Find  $dy/dx$  where  $\cos^3 x + \cos^3 y = \sin(x + y)$  (5%)

v. Find the differential  $dw$  where  $w = \exp(-x^3 - y^4)$ . (5%)

2. Evaluate the following integrals. (35%)

i.  $\int x\sqrt{x^2 + 25} dx$  (5%)    ii.  $\int \sin(\alpha x + \beta) dx$  (5%)    iii.  $\int \frac{e^{\sqrt{x}} dx}{\sqrt{x}}$  (5%)    iv.  $\int 3xe^{-2x} dx$  (5%)

v.  $\int \frac{dx}{(x+1)(x^2+1)}$  (5%)    vi.  $\int_1^{\infty} \frac{dx}{5x+1}$  (5%)    vii.  $\int_0^1 \int_{-2}^2 x^2 e^y dx dy$  (5%)

3. Evaluate the following limits. (15%)

i.  $\lim_{x \rightarrow 0^+} \frac{1 - \cos x}{x}$  (5%)    ii.  $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{2x}$  (5%)    iii.  $\lim_{x \rightarrow 0^+} (1 + \frac{1}{x})^x$  (5%)

4. Find the Taylor's 4<sup>th</sup> degree polynomial for  $f(x) = e^x$  at  $a=1$  with the remainder. (10%)

5. Graph the function  $f(x) = \frac{2x^2 + 1}{x^2 - 2x}$ . Please identify all extrema, inflection points, intercepts, and asymptotes. Show the concave structure and the behavior of the graph for  $|x|$  large and for  $x$  near any discontinuities of the function. (15%)