一、選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) Ninety students, including Vivien and Victoria, are to be split into three classes of equal size, and this is to be done at random. What is the probability that Vivien and Victoria end up in the same class?

(A) $\frac{29}{89}$

(B) $\frac{29}{267}$

(C) $\frac{1}{3}$

(D) $\frac{1}{30}$

(E) None of these

2. (5%) Suppose that $M(t)$ is a moment-generating of some random variable. Which of the following is also a moment-generating function of some random variable?

(A) $M(t) + M(5t)$

(B) $3M(t)$

(C) $e^{-t}M(t)$

(D) $\frac{M(t)}{t}$

(E) None of these

3. (5%) Assume that a random X satisfies
$$E[X] = 0, E[X^2] = 1, E[X^3] = 0, E[X^4] = 3,$$
and let
$$Y = 1 + X + X^2.$$
Which of the following is the correlation coefficient $\rho(X, Y)$?

(A) 0

(B) 1

(C) $\frac{1}{3}$

(D) $\frac{1}{\sqrt{3}}$

(E) None of these

4. (5%) A defective coin minting machine produces coins whose probability of heads is a random variable P with probability density function
$$f_P(p) = \begin{cases} pe^p, & p \in [0,1], \\ 0, & \text{otherwise.} \end{cases}$$
A coin produced by this machine is selected and tossed repeatedly, with successive tosses assumed independent. What is the probability that a coin toss results in heads?

(A) $\frac{1}{2}$

(B) p

(C) pe^p

(D) $e - 2$

(E) None of these

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5. (5%) Let X and Y be two random variables and $g(Y)$ be a function of Y. Assume that
$$E[Xg(Y)|Y] = g(Y)E[X|Y].$$
Which of the following statements is correct?
(A) X and Y are independent
(B) X and Y are uncorrelated
(C) $g(Y)$ is a constant
(D) $g(Y)$ is a linear function
(E) None of these

6. (5%) Let X be a random variable with probability distribution function given by
$$f(x) = \frac{1}{2}e^{-|x|}$$
Which of the following statements is wrong?
(A) $E[X] = 0$
(B) $Var(X) = 1$
(C) $P(|X| \leq u) = 1 - e^{-u}$
(D) The MGF of X is $M_X(t) = \frac{1}{1-t^2}$, for $|t| < 1$.
(E) None of these

7. (5%) Consider two random variables X and Y with joint probability mass function:

$P(x,y)$	$X = 1$	$X = 2$	$X = 3$
$Y = -1$	0.16	0.06	0.08
$Y = 0$	0.16	0.08	0.16
$Y = 1$	0.16	0.06	0.08

Which of the following statements is wrong?
(A) $E[X] = 1.84$
(B) $P(Y = 0) = 0.4$
(C) $E[XY] = 0$
(D) X and Y are uncorrelated
(E) X and Y are independent

8. (5%) Consider two random variables X and Y with joint probability distribution function $f(x,y) = ke^{-3x-2y}$, for $x \geq 0, y \geq 0$.
Which of the following statements is wrong?
(A) $k = 6$
(B) $E[Y] = 2$
(C) $E[XY] = 1/6$
(D) X and Y are uncorrelated
(E) X and Y are independent

9. (5%) Two fair and six-sided dies are rolled at the same time. Let A and B be the events:
　　　Event A : "Sum of two dies is four"
　　　Event B : "At least one die shows as one"
What is the conditional probability $P(B|A)$?
(A) 2/36
(B) 11/36
(C) 3/36
(D) 2/11
(E) 2/3

10. (5%) Let X be a random variable with probability distribution function given by

$$F(x) = \begin{cases} 0, & x < -2 \\ 0.3, & -2 \leq x < 0 \\ 0.7, & 0 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

Which of the following statements is wrong?

(A) $P(X = 0) = 0.4$

(B) $P(X > 1) = 0.3$

(C) $E[X] = 0$

(D) $Var(X) = 0.24$

(E) None of these

二、問答計算題:

1. (10%) The lifetimes of two light bulbs are modeled as independent and exponential random variables X and Y, with parameters λ and μ, respectively. The time at which a light bulb first burns out is
$$Z = \min\{X, Y\}.$$
Show that Z is an exponential random variable with parameter $\lambda + \mu$.

2. (15%) Let X and Y have joint probability density function given by $f(x,y) = \lambda^2 e^{-\lambda y}$ for $0 \leq x \leq y$, where $\lambda > 0$. Find the probability density function of $X + Y$.

3. (15%) Consider two discrete random variables X and . The random variable X has probability mass function
$$P(X = x) = \begin{cases} 0.4, & x = 1, \\ 0.6, & x = -1. \end{cases}$$
The conditional probability mass function of Y given X is given by:
$$P(Y = y | X = 1) = \begin{cases} 0.5, & y = 2 \\ 0.4, & y = 0 \\ 0.1, & y = -2 \end{cases}, \quad P(Y = y | X = -1) = \begin{cases} 0.2, & y = 2 \\ 0.3, & y = 0 \\ 0.5, & y = -2 \end{cases}$$

(a) (5%) Find the marginal probability distribution function of Y

(b) (5%) Find $E[XY]$. Are X and Y uncorrelated? Please explain your reason.

(c) (5%) Find the probability $P(X = 1 | Y = 0)$

4. (10%) Consider a random variable X with probability distribution function:
$$f(x) = xe^{-\frac{x^2}{2}}, \quad x \geq 0$$

(a) (5%) Find the cumulative distribution function of X.

(b) (5%) Let $Y = \frac{1}{2}X^2$. Find the probability distribution function of Y.