國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組、乙組選考】 題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

1. (10%) A double-sideband amplitude modulation (AM) transmitted signal can be expressed as

$$u(t)=2\left[1+\pi\sin 2\pi t\right]\cos\left(2\pi f_c t+\phi_c\right),$$

where f_c is the carrier frequency and ϕ_c is the phase. Can a simple envelope detector perfectly detect the message of $\pi\sin 2\pi t$? Explain your answer.

2. (10%) A binary PSK demodulation and detection receiver with a carrier phase error ϕ_e is considered, and the error probability can be expressed as

$$P\left(\phi_e\right)=Q\left(\sqrt{\frac{2E_b}{N_0}\cos^2\phi_e}\right),$$

where $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp\left(\frac{-u^2}{2}\right)du$. Assume that the probability density function (PDF) of ϕ_e is $p\left(\phi_e\right)=\frac{1}{\sqrt{2\pi}\sigma_{\phi_e}}e^{\frac{-\phi_e^2}{2\sigma_{\phi_e}^2}}$. Decide the expression for the average error probability in an integral form.

3. (25%) If two equiprobable messages of $s_1(t)$ and $s_2(t)$ are transmitted over an AWGN channel with the noise power spectral density of $N_0/2$, the error probability can be expressed as

$$P_e=Q\left(\sqrt{\frac{d^2}{2N_0}}\right),$$

where $d^2=\int_{-\infty}^{\infty}\left(s_1(t)-s_2(t)\right)^2dt$. Now, we further consider the case that $s_1(t)=u(t)$ and $s_2(t)=u(t-1)$, where $u(t)$ is shown in Fig. 1. Answer the following questions.

(a). (10%) Plot the diagram of an optimal matched filter receiver for the system. Explicitly label the required parameters.

(b). (5%) Decide the error probability for the system.

(c). (10%) Consider $s_1(t)=u(t)$ and $s_2(t)=\begin{cases}u(t-1) & \text{with probability } 0.5\\ u(t) & \text{with probability } 0.5\end{cases}$. Determine the optimum detection rule.

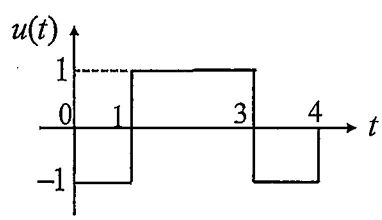

Fig. 1

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4. (20%) A carrier-modulated signal of $x(t) = a(t)\cos\left(w_c t + \theta(t)\right)$ can be transformed into an equivalent baseband signal, as shown in Fig. 2.

If $x(t) = \text{sinc}\left(10^5 t\right)\cos\left(2\pi 10^6 t\right) + \text{sinc}\left(10^5 t\right)\sin\left(2\pi 10^6 t\right)$, answer the following questions.

(a). (10%) Decide the values of w_c, $a(t)$, and $\theta(t)$.

(b). (10%) Determine $x_b(t)$ and $x_A(t)$

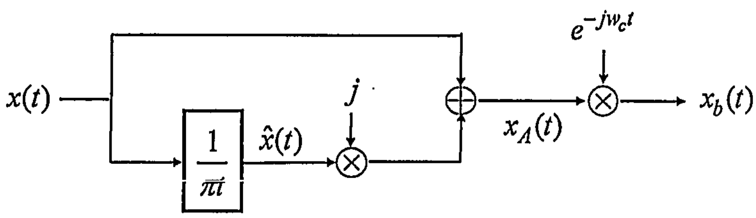

Fig. 2. Transform a passband signal to a baseband signal.

5. (15%) For a binary communication system shown in Fig. 3, the receiver obtains the two corrupted signals, y_1 and y_2, where two noises, n_1 and n_2, are not necessarily Gaussian distributions. The maximum a posteriori probability (MAP) receiver can be used to optimally detect the transmitted signal s from the observed signals, y_1 and y_2, i.e., $\hat{s} = \max_s p\left(s \mid y_1, y_2\right)$. If n_1 and n_2 are independent, can the optimum decision be based only on y_1 (i.e., $\hat{s} = \max_s p\left(s \mid y_1, y_2\right) = \max_s p\left(s \mid y_1\right)$)? Please justify your answer.

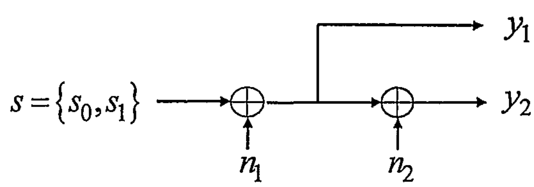

Fig. 3.

6. (20%) Denote $x(t)$ and $y(t)$ as the two bandpass real signals, and $x_L(t)$ and $y_L(t)$ are the corresponding lowpass equivalents with respect to the carrier frequency f_0. Thus, they have the following relations in frequency domain:

$$X(f)=\frac{1}{2}X_L(f-f_0)+\frac{1}{2}X_L^*(-f-f_0)$$

and

$$Y(f)=\frac{1}{2}Y_L(f-f_0)+\frac{1}{2}Y_L^*(-f-f_0),$$

where $X(f)$, $Y(f)$, $X_L(f)$, and $Y_L(f)$, are the frequency responses of $x(t)$, $y(t)$, $x_L(t)$, and $y_L(t)$, respectively.

(a). (15%) Assume $X_L(f-f_0)$ and $Y_L(-f-f_0)$ do not overlap. Show that $\int_{-\infty}^{\infty} x(t)y(t)dt=\frac{1}{2}\mathrm{Re}\left\{\int_{-\infty}^{\infty} x_L(t)y_L^*(t)dt\right\}$, where $\mathrm{Re}\{X\}$ represents the real part of the complex number X. Hint: Use Parseval's relation $\int_{-\infty}^{\infty} x(t)y(t)dt=\int_{-\infty}^{\infty} X(f)Y^*(f)df$.

(b). (5%) Prove that the energy in a bandpass signal is just one-half the energy in its lowpass equivalent.