國立中山大學107學年度碩士暨碩士專班招生考試試題

科目名稱：普通物理 【材光系碩士班丙組】　　題號：439003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(問答申論題)　　共2頁 第 1頁

Problem 1. [Mechanics: 35 points]

Materials are made of atoms linked by the electrostatic force. Around the equilibrium positions, the electrostatic force can be modeled as a spring with elastic constant k.

(a) [10 points] Consider 2 identical atoms with mass m, connected by a force modeled as a spring with elastic constant k. The center of mass is stationary (not moving). Assume the 2 atoms only move in x-direction, solve the angular frequency of the vibration motion of the 2 atoms. How many vibration modes?

(b) [5 points] While the atoms are vibrating, is the energy conserved? is the momentum conserved? is the angular momentum conserved (with respect to the center of mass)?

(c) [20 points] Now consider 2 identical particles with mass m, connected by a spring with elastic constance k. Each particle is attached to a wall with the spring (same elastic constant k). Assume the 2 atoms only move in x-direction, how many vibration modes? Also solve the angular frequencies of each vibration mode. (See the Figure 1 as an example. x_1 and x_2 are the displacements from the equilibrium positions for particle 1 and 2. Note that the figure is just a demonstration, not the solution)

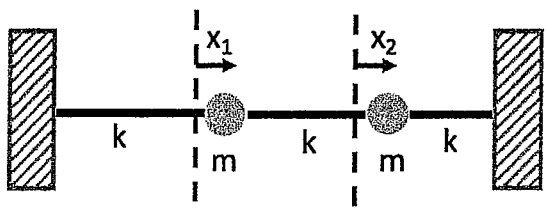

Figure 1

Problem 2. [Electromagnetism: 35 points]

Electromagnetic fields are the dominant forces in materials science. They are also directly related to optoelectronic applications. The electric field at a distance $\boldsymbol{r}$ from a point charge Q is:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\boldsymbol{r}},$$

where $\hat{\boldsymbol{r}}$ is the unit vector along the vector $\boldsymbol{r}$. Consider the following questions in the vacuum, and use the units in the above formula.

(a) [7 points] Find the electric field (magnitude and direction) a distance s away from the midpoint between two charges: $-q$ at $z = -d/2$ and q at $z = +d/2$.

(b) [7 points] The two equal and opposite charges in **(a)** is an electric dipole. Prove first the following approximation (hint: use Taylor's expansion):

$$(1+x)^\alpha \approx 1 + \alpha x, \quad \text{as } x \ll 1.$$

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Then use this approximation to calculate, when $s \ll d$, the electric field (the first two non-vanishing terms).

(c) [7 points] Find the capacitance of a spherical conductor of radius R, with total charge Q on its surface.

(d) [7 points] If we add a tiny charge dq to the surface of the conductor in **(c)** by bringing it from infinity, what is the work done if the charge is increased from 0 to Q. That is the energy stored in the capacitor.

(e) [7 points] Following **(d)**, prove

$$\text{Energy} = \frac{1}{2}\epsilon_0 \int |\mathbf{E}|^2 dv.$$

Problem 3. [Thermodynamics: 20 points]

(a) [10 points] An ideal gas of N molecules expands in such a way that the temperature of the gas remains constant T. Find the work done when the volume expands from V_1 to V_2. Boltzmann constant is k.

(b) [7 points] According the Maxwell-Boltzmann distribution, the number of particles n_i corresponding to the energy E_i in the most probable distribution at temperature T is given by:

$$n_i = A\, e^{f(E_i,T)},$$

where A is a constant. What is the function $f(E_i, T)$? (Boltzmann constant is k)

(c) [3 points] What is then the ratio of the occupation numbers at two energy levels E_i and E_j, that is, n_j/n_i?

Problem 4. [Waves/Optics: 10 points]

(a) [5 points] Let a string be stretched between two clamps separated by a fixed distance L (both ends are fixed). Find wave lengths of all possible standing waves.

(b) [5 points] If there are some plane wave $f(x,t)$, which follows the wave equation:

$$\frac{d^2 f}{dt^2} = \left(135.7 + \frac{T}{\rho - 1.05}\right)\frac{d^2 f}{dx^2},$$

where T is the tension and ρ is the density of the medium. What is the velocity of this wave?